Factoring polynomials will be the main topic on Exam 2; we will be factoring polynomials in Lessons 6 and 7, and factoring will come up again quite a bit in Lesson 8, somewhat in Lesson 9, and once or twice in Lesson 10. You will still need to understand the other topics that we will cover (such as solving linear equations in Lesson 10), but factoring polynomials will make-up about 75% of Exam 2, so please be prepared.

Throughout this course we will be looking at how to undo different operations in algebra. When covering exponents we showed how $(-3)^3 = -27$, then when covering radicals we saw how to get back to the original base of -3 by undoing an exponent of 3 with a cubed root $\left(\sqrt[3]{-27} = -3\right)$. When covering polynomial multiplication we showed how to multiply factors such as $(4x + 3)$ and $(x - 5)$ to obtain a product which is a polynomial $(4x^2 - 17x - 15)$, and in these notes we will show how to undo that product to get back to the original factors.

<u>Factoring Polynomials:</u>

- finding a product that is equivalent to some original polynomial
  - $4x^2 - 17x - 15$ is equivalent to $(4x + 3)(x - 5)$
- we will be using factoring as a way of undoing polynomial multiplication (going from a sum of terms to product of factors)
  - in the trinomial $4x^2 - 17x - 15$, $4x^2$, $-17x$, and -15 are all terms, while in the product $(4x + 3)(x - 5)$, $(x - 5)$ and $(4x + 3)$ are both factors
- not all polynomials are factorable
  - $x^2 + 2x + 3$ is an example of a polynomial that is prime, which means that trinomial cannot be expressed in factored form

<u>Greatest Common Factor (GCF):</u>

- the largest factor that is common to each term of an expression
- the GCF of an expression could be a number, a variable, a quantity, or some combine of the three
  - in the binomial $6x^2y^4 - 9x^3y^2$ the GCF is $3x^2y^2$
  - when the GCF is factored out, each term in the polynomial is divided by the $3x^2y^2$
    - $6x^2y^4 - 9x^3y^2 = 3x^2y^2\left(\frac{6x^2y^4}{3x^2y^2} - \frac{9x^3y^2}{3x^2y^2}\right)$

$$= \mathbf{3x^2y^2(2y^2 - 3x)}$$

Step one is identifying the GCF, and step two is dividing it out.

Example 1: Factor the following polynomials by taking out the GCF, and write each final answer in factored form.

a. $x^8 - x^3$

The GCF of x^8 and x^3 is x^3. Once you identify the GCF, the next step is to divide each term by the GCF.

$$x^3\left(\frac{x^8}{x^3} - \frac{x^3}{x^3}\right)$$

$$\boldsymbol{x^3(x^5 - 1)}$$

Keep in mind that the GCF is divided out, not subtracted off, so that is why we end up with $x^3(x^5 - 1)$ and **NOT** $x^3(x^5 - 0)$.

b. $2x^4y^3 - 8x^5y^6$

$$2x^4y^3\left(\frac{2x^4y^3}{2x^4y^3} - \frac{8x^5y^6}{2x^4y^3}\right)$$

$$\boldsymbol{2x^4y^3(1 - 4xy^3)}$$

c. $-33x^9y^3z^6 + 15x^5y^6z^4 - 24x^7y^9z^8$

$$3x^5y^3z^4\left(\frac{-33x^9y^3z^6}{3x^5y^3z^4} + \frac{15x^5y^6z^4}{3x^5y^3z^4} - \frac{24x^7y^9z^8}{3x^5y^3z^4}\right)$$

$$\boldsymbol{3x^5y^3z^4(-11x^4z^2 + 5y^3 - 8x^2y^6z^4)}$$

The GCF of an expression does not have to be simply a number or a variable. As stated before, the GCF could be a quantity as well, as we'll see in the next example:

- in the trinomial $3x(x-1)^2 - 5(x-1)^3 + 8y(x-1)$ the GCF is $(x-1)$
- when the GCF is factored out, each term in the polynomial is divided by the $(x-1)$

$$3x(x-1)^2 - 5(x-1)^3 + 8y(x-1)$$

$$(x-1)\left(\frac{3x(x-1)^2}{(x-1)} - \frac{5(x-1)^3}{(x-1)} + \frac{8y(x-1)}{(x-1)}\right)$$

$$(x-1)(3x(x-1) - 5(x-1)^2 + 8y)$$

Once the GCF of $(x-1)$ is factored out, we cannot factor the remaining polynomial any further. So we go back to what we did in the previous lesson; multiply the polynomials and combine like terms.

$$(x-1)(3x^2 - 3x - 5(x-1)(x-1) + 8y)$$

$$(x-1)(3x^2 - 3x - 5(x^2 - 2x + 1) + 8y)$$

$$(x-1)(3x^2 - 3x - 5x^2 + 10x - 5 + 8y)$$

$$\boldsymbol{(x-1)(-2x^2 + 7x + 8y - 5)}$$

After distributing factors and combining like terms, the resulting polynomial can sometimes be factored further. That is not the case on this problem, but it could be with other problems.

Keep in mind that when a GCF is factored out, we don't list it more than once. For instance when $(y+1)$ is factored out of the binomial $3x(y+1) - 4(y+1)$, we have $(y+1)(3x-4)$, **<u>NOT</u>**

$(y+1)(y+1)(3x-4)$. The same is true if we factor a y from the binomial $3xy-4y$ to get $y(3x-4)$; both terms had a common factor of y, but when we factor it out we only have one factor of y as the GCF.

<u>Example 2:</u> Factor the following polynomials by taking out the GCF, and write each final answer in factored form.

a. $(x+1)(x-2)+(x+1)(x+3)$

$$(x+1)\left(\frac{(x+1)(x-2)}{(x+1)}+\frac{(x+1)(x+3)}{(x+1)}\right)$$

$$(x+1)\big((x-2)+(x+3)\big)$$

$$(x+1)(x-2+x+3)$$

$$\boldsymbol{(x+1)(2x+1)}$$

b. $(x+y)(x-9)-(9x-1)(y+x)$

$$(x+y)\left(\frac{(x+y)(x-9)}{(x+y)}-\frac{(9x-1)(y+x)}{(x+y)}\right)$$

$$(x+y)\big((x-9)-(9x-1)\big)$$

$$(x+y)(x-9-9x+1)$$

$$(x+y)(-8x-8)$$

$$(x+y)(-8)(x+1)$$

$$\boldsymbol{-8(x+y)(x+1)}$$

Keep in mind that the binomial $(9x-1)$ must have parentheses around it so that the negative sign in front of it gets distributed to both terms ($9x$ and -1). Had there been a plus sign in front of the binomial $9x-1$, parentheses would not have been required, but they could still be included just for consistency.

c. $2(1-x)^3-3x(1-x)^2$

$$(1-x)^2\left(\frac{2(1-x)^3}{(1-x)^2}-\frac{3x(1-x)^2}{(1-x)^2}\right)$$

$$(1-x)^2(2(1-x)-3x)$$

$$(1-x)^2(2-2x-3x)$$

$$\boldsymbol{(1-x)^2(2-5x)}$$

d. $4x^2(x-1)^2 - 18x(x-1)(3x+2)$

$$2x(x-1)\left(\frac{4x^2(x-1)^2}{2x(x-1)} - \frac{18x(x-1)(x+2)}{2x(x-1)}\right)$$

$$2x(x-1)\big(2x(x-1) - 9(x+2)\big)$$

$$2x(x-1)(2x^2 - 2x - 9x - 18)$$

$$\boldsymbol{2x(x-1)\big(2x^2 - 11x - 18\big)}$$

e. $4x^4(x-1)^2 - 6x^2(x-1)^3 + 8x^3(x-1)$

$$2x^2(x-1)\left(\frac{4x^4(x-1)^2}{2x^2(x-1)} - \frac{6x^2(x-1)^3}{2x^2(x-1)} + \frac{8x^3(x-1)}{2x^2(x-1)}\right)$$

$$2x^2(x-1)(2x^2(x-1) - 3(x-1)^2 + 4x)$$

$$2x^2(x-1)(2x^3 - 2x^2 - 3(x-1)(x-1) + 4x)$$

$$2x^2(x-1)(2x^3 - 2x^2 - 3(x^2 - 2x + 1) + 4x)$$

$$2x^2(x-1)(2x^3 - 2x^2 - 3x^2 + 6x - 3 + 4x)$$

$$\boldsymbol{2x^2(x-1)\big(2x^3 - 5x^2 + 10x - 3\big)}$$

Answers to Examples:

1a. $x^3(x^5-1)$; *1b.* $2x^4y^3(1-4xy^3)$;

1c. $3x^5y^3z^4(-11x^4z^2 + 5y^3 - 8x^2y^6z^4)$; *2a.* $(x+1)(2x+1)$;

2b. $-8(x+y)(x+1)$ *; 2c.* $(1-x)^2(2-5x)$ *;*

2d. $2x(x-1)(2x^2-29x-18)$ *;*

2e. $2x^2(x-1)(-x^3+4x^2+x)$ *;*