Rational expression:

- a fraction with polynomials in the numerator and denominator (also called a quotient or ratio of polynomials)
  - $\frac{x+2}{x^2-5x-6}$ is an example of a rational expression; the numerator is a binomial and the denominator is a trinomial

Since a rational expression is simply a fraction, it can be simplified just like any other fraction by factoring the numerator and denominator and then canceling any common factors that they may have.

Example 1: Simplify the following fractional expressions completely.

a. $\frac{6}{8}$

b. $\frac{14x^4y}{18x^3y^2}$

$\frac{2 \cdot 7 \cdot x \cdot x \cdot x \cdot x \cdot y}{2 \cdot 9 \cdot x \cdot x \cdot x \cdot y \cdot y}$

$\frac{\cancel{2} \cdot 7 \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot \cancel{y}}{\cancel{2} \cdot 9 \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot y \cdot \cancel{y}}$

$\frac{7 \cdot x}{9 \cdot y}$

$\boldsymbol{\frac{7x}{9y}}$

Steps for Simplifying Rational Expressions:

1. remove parentheses and combine like terms (if necessary)
2. factor all the polynomials
3. cancel common factors

A rational expression is simplified if its numerator and denominator have no common factors other than 1, just like the rational expression $\frac{x-2}{x+2}$.

It is imperative that you understand how to factor polynomials prior to simplify rational expressions. Keep in mind that rational expressions are simply fractions, and just like any other type of fraction, they should be simplified completely by canceling common factors.

<u>Example 2:</u> Simplify the rational expressions completely.

a. $\frac{8+x^3}{x^4-16}$

$\frac{2\cdot2\cdot2+x\cdot x\cdot x}{x^2\cdot x^2-4\cdot4}$

$\frac{(2+x)\left(2^2-(2)(x)+x^2\right)}{(x^2-4)(x^2+4)}$

$\frac{(2+x)\left(4-2x+x^2\right)}{(x\cdot x-2\cdot2)(x^2+4)}$

$\frac{(2+x)\left(4-2x+x^2\right)}{(x-2)(x+2)(x^2+4)}$

$\frac{\cancel{(2+x)}\left(4-2x+x^2\right)}{(x-2)\cancel{(x+2)}(x^2+4)}$

$\mathbf{\frac{x^2-2x+4}{(x-2)(x^2+4)}}$

b. $\frac{12x^4-17x^2+5}{8x^9-8x^8}$

c. $\frac{3x^5-26x^4-40x^3}{\pi x^6-100\pi x^4}$

d. $\frac{3+13x-10x^2}{25x^2-1}$

$\frac{-1\left(10x^2-13x-3\right)}{(5x)^2-(1)^2}$

$\frac{-1\left(10x^2+2x-15x-3\right)}{(5x+1)(5x-1)}$

$\frac{-1\left(2x(5x+1)-3(5x+1)\right)}{(5x+1)(5x-1)}$

$\frac{-1(5x+1)(2x-3)}{(5x+1)(5x-1)}$

$\frac{-1(\cancel{5x+1})(2x-3)}{(\cancel{5x+1})(5x-1)}$

$\mathbf{\frac{3-2x}{5x-1}}$

e. $\frac{(x+h)^2-2(x+h)+1-\left(x^2-2x+1\right)}{h}$

f. $\frac{(x+h)^3+x+h-\left(x^3+x\right)}{h}$

On each of these rational expression, we need to remove the parentheses from the numerator first, and then combine like terms. To remove parentheses we use polynomial multiplication, and to combine like terms we use polynomial addition (both **Lesson 5** topics). While these are both rational expressions, they are also both problems that can be simplified completely using techniques from **Lesson 5**.

$$\frac{(x+h)^2-2(x+h)+1-\left(x^2-2x+1\right)}{h}$$

$$\frac{(x+h)(x+h)-2x-2h+1-x^2+2x-1}{h}$$

$$\frac{x^2+2xh+h^2-2x-2h+1-x^2+2x-1}{h}$$

Once all the parentheses have been removed, the like terms can be combined. In this case the like terms also end up being opposite terms.

$$\frac{\cancel{x^2}+2xh+h^2\cancel{-2x}-2h\cancel{+1}\cancel{-x^2}\cancel{+2x}\cancel{-1}}{h}$$

$$\frac{2xh+h^2-2h}{h}$$

$$\frac{2xh}{h}+\frac{h^2}{h}-\frac{2h}{h}$$

$$\mathbf{2x+h-2}$$

$$\frac{(x+h)^3+x+h-\left(x^3+x\right)}{h}$$

$$\frac{(x+h)(x+h)(x+h)+x+h-x^3-x}{h}$$

$$\frac{\left(x^2+2xh+h^2\right)(x+h)+x+h-x^3-x}{h}$$

$$\frac{x^3+x^2h+2x^2h+2xh^2+xh^2+h^3+x+h-x^3-x}{h}$$

Once all the parentheses have been removed, the like terms can be combined. In this case the like terms also end up being opposite terms.

$$\frac{\cancel{x^3}+x^2h+2x^2h+2xh^2+xh^2+h^3\cancel{+x}+h\cancel{-x^3}\cancel{-x}}{h}$$

$$\frac{3x^2h+3xh^2+h^3+h}{h}$$

$$\frac{3x^2h}{h}+\frac{3xh^2}{h}+\frac{h^3}{h}+\frac{h}{h}$$

$$\mathbf{3x^2+3xh+h^2+1}$$

Answers to Examples:

1a. $\frac{3}{4}$; *1b.* $\frac{9x^2}{16y}$; *2a.* $\frac{x^2-2x+4}{(x-2)(x^2+4)}$; *2b.* $\frac{(12x^2-5)(x+1)}{8x^8}$; *2c.* $\frac{3x+4}{\pi x(x+10)}$;
2d. $\frac{-1(2x-3)}{5x-1}$; *2e.* $2x+h-2$; *2f.* $3x^2+3xh+h^2+1$;