**Rational expression:**
- a fraction with polynomials in the numerator and denominator (also called a quotient or ratio of polynomials)
  - $\frac{x+2}{x^2-5x-6}$ is an example of a rational expression; the numerator is a binomial and the denominator is a trinomial

Since a rational expression is simply a fraction, it can be simplified just like any other fraction by factoring the numerator and denominator and then canceling any common factors that they may have.

**Example 1:** Simplify the following fractional expressions completely.

a. $\frac{6}{8}$

b. $\frac{14x^4y}{18x^3y^2}$

\[
\begin{align*}
6 &= 2 \cdot 3 \\
8 &= 2^3 \\
14x^4y &= 2 \cdot 7 \cdot x \cdot x \cdot x \cdot y \\
18x^3y^2 &= 2 \cdot 9 \cdot x \cdot x \cdot x \cdot y \cdot y \\
\frac{2\cdot7\cdot x \cdot x \cdot x \cdot y}{2\cdot9\cdot x \cdot x \cdot x \cdot y \cdot y} &= \frac{7\cdot x}{9\cdot y} \\
\frac{7x}{9y} &= \frac{7x}{9y}
\end{align*}
\]

**Steps for Simplifying Rational Expressions:**
1. remove parentheses and combine like terms (if necessary)
2. factor all the polynomials
3. cancel common factors

A rational expression is simplified if its numerator and denominator have no common factors other than 1, just like the rational expression $\frac{x-2}{x+2}$.

It is imperative that you understand how to factor polynomials prior to simplify rational expressions. Keep in mind that rational expressions are simply fractions, and just like any other type of fraction, they should be simplified completely by canceling common factors.
Example 2: Simplify the rational expressions completely.

a. \( \frac{8 + x^3}{x^4 - 16} \)

b. \( \frac{12x^4 - 17x^2 + 5}{8x^9 - 8x^8} \)

c. \( \frac{3x^5 - 26x^4 - 40x^3}{\pi x^6 - 100\pi x^4} \)

d. \( \frac{3 + 13x - 10x^2}{25x^2 - 1} \)

\[
\begin{align*}
&= \frac{-1(10x^2 - 13x - 3)}{(5x)^2 - (1)^2} \\
&= \frac{-1(10x^2 + 2x - 15x - 3)}{(5x + 1)(5x - 1)} \\
&= \frac{-1(2x(5x + 1) - 3(5x + 1))}{(5x + 1)(5x - 1)} \\
&= \frac{-1(5x + 1)(2x - 3)}{(5x + 1)(5x - 1)} \\
&= \frac{3 - 2x}{5x - 1}
\end{align*}
\]
On this rational expression we need to remove the parentheses and combine like terms first. To remove parentheses I will use polynomial multiplication, and to combine like terms I will use polynomial addition (both Lesson 5 topics).

\[
\frac{(x+h)^2-2(x+h)+1-(x^2-2x+1)}{h}
\]

\[
\frac{(x+h)(x+h)-2x-2h+1-x^2+2x-1}{h}
\]

\[
\frac{x^2+2hx+h^2-2x-2h+1-x^2+2x-1}{h}
\]

Once all the parentheses have been removed, the like terms can be combined. In this case the like terms also end up being opposite terms.

\[
\frac{2hx+h^2-2h}{h} + \frac{h^2}{h} - \frac{2h}{h}
\]

\[
2x + h - 2
\]
Answers to Examples:

1a. $\frac{3}{4}$; 1b. $\frac{9x^2}{16y}$; 2a. $\frac{x^2-2x+4}{(x-2)(x^2+4)}$; 2b. $\frac{(12x^2-5)(x+1)}{8x^8}$; 2c. $\frac{3x+4}{\pi(x+10)}$

2d. $\frac{-1(2x-3)}{5x-1}$; 2e. $2x + h - 2$; 2f. $3x^2 + 3xh + h^2 + 1$