Rational expression:
- a fraction with polynomials in the numerator and denominator
  o \( \frac{x^2-6x}{x^2-5x-6} \) is an example of a rational expression; the numerator is a binomial and the denominator is a trinomial
  o \( \frac{x^2-6x}{x^2-5x-6} \) can be simplified by factoring the numerator and denominator and canceling the common factors
    ▪ \( \frac{x^2-6x}{x^2-5x-6} \) factors as \( \frac{x(x-6)}{(x+1)(x-6)} \), which simplifies to \( \frac{x}{x+1} \) when the common factor of \( (x-6) \) is canceled from the numerator and denominator
    ▪ keep in mind that only FACTORS are canceled, not terms, so \( \frac{x}{x+1} \) cannot be reduced any further because the factor of \( x \) in the numerator and the term of \( x \) in the denominator do not cancel

Steps for Simplifying Rational Expressions:
1. remove parentheses and combine like terms (if necessary)
2. factor all the polynomials
3. cancel common factors

In addition to simplifying rational expressions, we also covered multiplying rational expressions in the previous lesson.

Steps for Multiplying Rational Expressions:
1. write numerator times numerator and denominator times denominator, but do not actually multiply the polynomials (leave in factored form)
2. factor the numerator and denominator completely, then cancel common factors (if possible)

It is imperative that you understand how to simplify and multiply rational expressions prior to dividing rational expressions.
**Example 1:** Divide the following fractions and simplify your answers completely.

a. \[ \frac{21}{20} \div \frac{35}{12} \]

b. \[ \frac{4a}{15} \div \frac{16}{25b} \div \frac{abc}{100} \]

\[ \left( \frac{4a}{15} \cdot \frac{25b}{16} \right) \div \frac{abc}{100} \]

\[ \left( \frac{4 \cdot a \cdot 5 \cdot 5 \cdot b}{3 \cdot 5 \cdot 4 \cdot 4} \right) \div \frac{abc}{100} \]

\[ \left( \frac{5ab}{12} \right) \div \frac{abc}{100} \]

\[ \frac{5ab}{12} \cdot \frac{100}{abc} \]

\[ \frac{5 \cdot a \cdot b \cdot 4 \cdot 25}{3 \cdot 4 \cdot a \cdot b \cdot c} \]

\[ \frac{125}{3c} \]

When dividing rational expressions, or any type of fractions, we don’t actually divide anything. Instead we change division to multiplication by taking the reciprocal of the divisor, and repeat the same steps as above.

**Steps for Dividing Rational Expressions:**
1. take the reciprocal of the divisor and change division to multiplication
2. write numerator times numerator and denominator times denominator, but do not actually multiply the polynomials (leave in factored form)
3. factor the numerator and denominator completely, then cancel common factors (if possible)
Example 2: Divide the following rational expressions and simplify your answers completely.

a. \( \frac{x^2-x-6}{x^2+6x+9} \div \frac{x^2-4}{x+3} \)

b. \( \frac{9x^2-1}{3x^2+5x-2} \div \frac{2x-1}{6x^2+17x+10} \)
c. \[
\frac{x^3 - 25x}{4x^2} \cdot \frac{2x^2 - 2}{x^2 - 6x + 5} \div \frac{x^2 + 5x}{7x+7}
\]

d. \[
\frac{x^2 + 5x + 6}{x^2 - 2x - 3} \div \frac{x+3}{x^2+7x+6} \cdot \frac{x^2 + x - 12}{x^2 + x - 30}
\]

Answers to Examples:

1a. \[
\frac{9}{25} ; 1b. \frac{125}{3c} ; 2a. \frac{(x-3)}{(x+3)(x-2)} ; 2b. \frac{(3x+1)(6x+5)}{(2x-1)} ; 2c. \frac{7(x+1)^2}{2x^2} ;
\]
2d. \[
\frac{(x+2)(x+4)}{x-5}
\]