Example 1: Divide the following fractions and simplify your answers completely.

a. $\frac{21}{20} \div \frac{35}{12}$

b. $\frac{4a}{15} \div \frac{16}{25b} \div \frac{abc}{100}$

$$\left(\frac{4a}{15} \cdot \frac{25b}{16}\right) \div \frac{abc}{100}$$

$$\left(\frac{4 \cdot a \cdot 5 \cdot 5 \cdot b}{3 \cdot 5 \cdot 4 \cdot 4}\right) \div \frac{abc}{100}$$

$$\left(\frac{5ab}{12}\right) \div \frac{abc}{100}$$

$$\frac{5ab}{12} \cdot \frac{100}{abc}$$

$$\frac{5 \cdot a \cdot b \cdot 4 \cdot 25}{3 \cdot 4 \cdot a \cdot b \cdot c}$$

$$\mathbf{\frac{125}{3c}}$$

When you're given more than two fractions to divide, simply work with just two fractions as at time, going from left to right.

Remember that to change the division sign to multiplication you take the reciprocal of the divisor (the fraction that appears to the right of the division sign).

Also keep in mind that when multiplying fractions, it's best to keep everything in factored form rather than to actually multiply the factors out, since our goal with any fraction is to factor and cancel common factors.

When dividing rational expressions, or any type of fractions, we don't actually divide anything. Instead we change division to multiplication by taking the reciprocal of the divisor, and repeat the same steps as above.

Steps for Dividing Rational Expressions:

1. take the reciprocal of the divisor and change division to multiplication
2. write numerator times numerator and denominator times denominator, but do not actually multiply the polynomials (**leave in factored form**)
3. factor the numerator and denominator completely, then cancel common factors (if possible)

<u>Example 2:</u> Divide the following rational expressions and simplify your answers completely.

a. $\frac{x^2-x-6}{x^2+6x+9} \div \frac{x^2-4}{x+3}$

b. $\frac{9x^2-1}{3x^2+5x-2} \div \frac{2x-1}{6x^2+17x+10}$

$$\frac{9x^2-1}{3x^2+5x-2} \cdot \frac{6x^2+17x+10}{2x-1}$$

$$\frac{\left((3x)^2-(1)^2\right)\left(6x^2+5x+12x+10\right)}{(3x^2+6x-1x-2)(2x-1)}$$

$$\frac{(3x-1)(3x+1)\left(x(6x+5)+2(6x+5)\right)}{\left(3x(x+2)-1(x+2)\right)(2x-1)}$$

$$\frac{(3x-1)(3x+1)(6x+5)(x+2)}{(x+2)(3x-1)(2x-1)}$$

$$\frac{\cancel{(3x-1)}(3x+1)(6x+5)\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(3x-1)}(2x-1)}$$

$$\boldsymbol{\frac{(3x+1)(6x+5)}{2x-1}}$$

Example 3: Combine the following rational expressions using the given operations, and simplify your answers completely. Keep in mind order of operation (PEMDAS).

a. $\frac{x^3-25x}{4x^2} \cdot \frac{2x^2-2}{x^2-6x+5} \div \frac{x^2+5x}{7x+7}$

$\frac{(x^3-25x)(2x^2-2)}{4x^2(x^2-6x+5)} \div \frac{x^2+5x}{7x+7}$

$\frac{x(x^2-25)(2)(x^2-1)}{4x^2(x^2-1x-5x+5)} \div \frac{x^2+5x}{7x+7}$

$\frac{x(x-5)(x+5)(2)(x-1)(x+1)}{4x^2(x-1)(x-5)} \div \frac{x^2+5x}{7x+7}$

$\frac{(x+5)(x+1)}{2x} \div \frac{x^2+5x}{7x+7}$

$\vdots$

b. $\frac{x^2+5x+6}{x^2-2x-3} \div \frac{x+3}{x^2+7x+6} \cdot \frac{x^2+x-12}{x^2+x-30}$

$\left(\frac{x^2+5x+6}{x^2-2x-3} \cdot \frac{x^2+7x+6}{x+3}\right) \cdot \frac{x^2+x-12}{x^2+x-30}$

$\frac{(x+2)(x+3)(x+1)(x+6)}{(x-3)(x+1)(x+3)} \cdot \frac{x^2+x-12}{x^2+x-30}$

$\frac{(x+2)(x+6)}{x-3} \cdot \frac{x^2+x-12}{x^2+x-30}$

$\vdots$

Answers to Examples:

1a. $\frac{9}{25}$ *; 1b.* $\frac{125}{3c}$ *; 2a.* $\frac{(x-3)}{(x+3)(x-2)}$ *; 2b.* $\frac{(3x+1)(6x+5)}{(2x-1)}$ *; 3a.* $\frac{7(x+1)^2}{2x^2}$ *;*

3b. $\frac{(x+2)(x+4)}{x-5}$ *;*