**Rational expression:**
- a fraction with polynomials in the numerator and denominator
  \( \frac{3x^5 - 26x^4 - 40x^3}{\pi x^6 - 100\pi x^4} \) is an example of a rational expression; the numerator is a trinomial and the denominator is a binomial

**Steps for Simplifying Rational Expressions:**
1. remove parentheses and combine like terms (if necessary)
2. factor all the polynomials
3. cancel common factors

**Once again, it is imperative that you understand how to factor polynomials prior to simplify rational expressions. Keep in mind that rational expressions are simply fractions, and just like any other type of fraction, they should be simplified completely by canceling common factors.

**Example 1:** Simplify the rational expressions completely.

a. \( \frac{3x^5 - 26x^4 - 40x^3}{\pi x^6 - 100\pi x^4} \)

b. \( \frac{3+13x-10x^2}{25x^2-1} \)

\[
\frac{-1(10x^2-13x-3)}{(5x)^2-(1)^2}
\]

\[
\frac{-1(10x^2+2x-15x-3)}{(5x+1)(5x-1)}
\]

\[
\frac{-1(2x(5x+1)-3(5x+1))}{(5x+1)(5x-1)}
\]

\[
\frac{-1(5x+1)(2x-3)}{(5x+1)(5x-1)}
\]

\[
\frac{-1(5x+1)(2x-3)}{(5x+1)(5x-1)}
\]

\[
3-2x
\]

\[
5x-1
\]
c. \( \frac{3a^4 + 2a^3 - 3a - 2}{27a^4 + 8a} \)

d. \( \frac{x^8 - 256}{x^4 + x^2 - 20} \)
Example 2: Simplify the rational expressions completely.

a. \[
\frac{(x+h)^2-2(x+h)+1-(x^2-2x+1)}{h}
\]

b. \[
\frac{(x+h)^3+x+h-(x^3+x)}{h}
\]

On each of these rational expression, we need to remove the parentheses from the numerator first, and then combine like terms. To remove parentheses we use polynomial multiplication, and to combine like terms we use polynomial addition (both Lesson 5 topics). While these are both rational expressions, they are also both problems that can be simplified completely using techniques from Lesson 5.

\[
\frac{(x+h)^2-2(x+h)+1-(x^2-2x+1)}{h} = \frac{(x+h)(x+h)-2x-2h+1-x^2+2x-1}{h} = \frac{x^2+2xh+h^2-2x-2h+1-x^2+2x-1}{h} = \frac{2xh+h^2-2h}{h} = \frac{2xh}{h} + \frac{h^2}{h} - \frac{2h}{h} = 2x + h - 2
\]

\[
\frac{(x+h)^3+x+h-(x^3+x)}{h} = \frac{(x+h)(x+h)(x+h)+x+h-x^3-x}{h} = \frac{(x^2+2xh+h^2)(x+h)+x+h-x^3-x}{h} = \frac{x^3+2x^2h+2xh^2+hx^2+h^3+x+h-x^3-x}{h} = \frac{3x^2h+3xh^2+h^3+h}{h} = \frac{3x^2h}{h} + \frac{3xh^2}{h} + \frac{h^3}{h} + \frac{h}{h} = 3x^2 + 3xh + h^2 + 1
\]
c. \[
\frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}
\]

d. \[
\frac{(x+h)^3 + 5(x+h) - (x^3 + 5x)}{h}
\]
Answers to Examples:

1a. \( \frac{3x+4}{\pi x(x+10)} \); 1b. \( \frac{-1(2x-3)}{5x-1} \); 1c. \( \frac{(a-1)(a^2+a+1)}{a(9a^2-6a+4)} \); 1d. \( \frac{(x^2+4)(x^4+16)}{x^2+5} \);

2a. \( 2x + h - 2 \); 2b. \( 3x^2 + 3x + h^2 + 1 \); 2c. \( 2x + h - 3 \); 2d. \( 3x^2 + 3x + h^2 + 5 \);