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EPR: how subtle is the Lord and how is the Lord subtle?

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The article offers a counterargument to the argument of A. Einstein, B. Podolsky and N. Rosen (EPR) concerning the incompleteness, or else nonlocality, of quantum mechanics, based on Bohr's reply to EPR's article. The article also relates argument to the impossibility of exact repetition of quantum events.

1. Introduction

The current stage of the debates concerning quantum mechanics is dominated by the arguments addressing quantum entanglement and, correspondingly, a particular (inherently quantum) type of correlations between certain spatially separated quantum events. These correlations are properly predicted by quantum mechanics but can be ascertained experimentally regardless of any theory. These arguments were initiated by the article of A. Einstein, B. Podolsky and N. Rosen (EPR), and by N. Bohr's reply to it, both published under the same title, 'Can Quantum-Mechanical Description of Physical Reality be Considered Complete?', in 1935 [1]; see also [2, v. 4, pp. 73–82]. As the subject developed, the question of the locality (compatibility with relativity) of quantum phenomena or quantum mechanics, rather than of its completeness, took centre stage. Some see these correlations as implying a violation of either or both types of locality. EPR's argument and related arguments by Einstein contend that quantum mechanics is either incomplete or nonlocal [1, p. 141]. Einstein dubbed this nonlocality 'a spooky action at a distance'. Bohr contests this argumentation and offers an interpretation of quantum phenomena and quantum mechanics, known as complementarity, according to which 'quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands for completeness', without sacrificing locality [2, v. 4, p. 74]. Bohr argues that quantum phenomena, including the EPR type, disallow EPR's conception of physical reality and the corresponding criterion of physical reality, or at least, the unqualified way in which the criterion is used by EPR. Hence, he argues, rather than demonstrating a deficiency of quantum mechanics with respect to either its completeness or its locality, *EPR's argument* concerning *the EPR experiment* is made insufficient by the nature

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of quantum phenomena. (I distinguish *the EPR experiment*, the thought experiment EPR consider, from *EPR's argument* concerning it.) Ironically, any classical-like theory of the type Einstein hoped for that would predict these correlations or quantum data in general appears to be nonlocal in view of Bell's and related theorems (e.g. the Kochen–Specker theorem), although these questions are subjects of continuing debates. The counterargument Bohr had in mind proved to be difficult to formulate, as he acknowledged [2, v. 2, p. 61]. Indeed it does not appear that Bohr ever published a completely worked-through counterargument. Bohr's reply to EPR, however, supplies just about all of the ingredients necessary for formulating such a counterargument, which this article will attempt to do.

2. Can quantum mechanics be considered complete?

EPR base their argument on two key criteria. One is a (necessary) criterion of completeness: for a theory to be considered complete, '*every element of the physical reality must have a counterpart in the physical theory*'. The other is a (sufficient) criterion of reality: '*If, without in any way disturbing a system, we can predict with certainty (i.e. with the probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*' [1, p. 138]. EPR's argument for the incompleteness of quantum mechanics in accordance with the first criterion depends on the postulate, assumed by EPR as self-evident, that the second criterion applies in quantum physics as it does in classical physics. Bohr argues, however, that, while EPR's criterion of reality does unambiguously apply in classical physics, it acquires 'an essential ambiguity' when it is applied without further qualifications (not offered by EPR) to quantum phenomena. Once this ambiguity is removed, it can be shown that every element of reality that can be ascertained for a well-defined quantum phenomenon does have a counterpart in quantum mechanics, insofar as it can be predicted by the latter. (The probability of such predictions is not always unity, although sometimes it is, as in the EPR experiment.) This, however, appears to be as much as nature allows us, which makes quantum mechanics as complete as any theory of quantum phenomena could be.

According to Bohr, the fundamental difference between quantum and classical phenomena arises by virtue of the irreducible role of measuring instruments in the constitution of quantum phenomena, and, as a result, the impossibility of considering the behaviour of quantum objects independently of their interaction with these instruments [2, v. 2, p. 61]. The EPR experiment allows for predictions (with certainty) concerning quantum objects without physically interfering with them by means of measurement. These predictions are enabled by performing measurements on other quantum objects, which have previously been in interaction with the objects in question, but are, at the time of such measurements, spatially separated from them. This circumstance leads EPR to argue, on the basis of their criterion, that the quantities thus *predicted* may be attributed to quantum objects themselves and, hence, that one need not involve measuring instruments in determining these quantities. This in turn compels them to argue that quantum objects independently possess both quantities in question (de facto circumventing the uncertainty relations), even though we can never predict or measure both simultaneously. Bohr counterargues that the situation does not allow one to dispense with the irreducible role of measuring instruments, since this role entails limitations on the *types* of measuring arrangements used in determining the quantities in question, even if one does so in terms of prediction rather than measurement. This fact, according to Bohr, disables EPR's argument by making EPR's use of their criterion of reality in their argument ambiguous.

Bohr's thinking concerning the situation eventually led him to an interpretation of quantum phenomena and quantum mechanics in which *only what has already occurred* determines any physical quantity considered, and *not what has been predicted* (even with certainty) and is yet to be confirmed by a measurement. Moreover, all measurable quantities are now seen as pertaining strictly to phenomena in Bohr's special sense of the term, defined in terms of the effects of the interactions between quantum objects and measuring instruments upon these instruments, say, a change in momentum of a part of a measuring instrument under the impact of a quantum object. Those parts of measuring instruments where such effects are manifest are described by means of classical physics, while those parts that interact with quantum objects are seen as quantum. By contrast, an attribution even of a single property to a quantum object is *never possible—before, during, or after measurement*. In this view, the mathematical formalism of quantum mechanics does not describe any physical processes but only offers predictive algorithms for the outcomes of certain experiments. EPR's criterion of reality no longer applies, although one can still make EPR-type *predictions* concerning future interactions between quantum objects and measuring devices without a disturbance or interference—*a spooky prediction at a distance*, but *without a spooky action at a distance*. Such predictions remain 'spooky' because there is still a question of how they are possible (i.e. what are the physical processes responsible for them). Bohr's answer is that we do not and may never know how this is possible.

Bohr's later view, just sketched, acquires new significance in view of the experiments reported in [3, 4]. The experiments reported in [3] indicate an apparent inapplicability of EPR's criterion of reality for photon pairs produced in parametric down conversion, which approximate the idealized entangled state constructed by EPR for continuous variables. (The original EPR state is an idealization and cannot be realized in actual experiments.) At the same time, we can rigorously distinguish between classical and quantum correlations in the case of continuous variables in different experiments approximating the EPR states [4]. These important experiments are relevant to my argument, especially given that they deal with the continuous variables considered by EPR rather than with discrete variables, as has been more common in the wake of Bell's theorem. But their significance in the present context, defining by Bohr's reply to ERP, requires a detailed discussion, especially as concerns their important statistical aspects, which, unfortunately, cannot be pursued here.

Unlike Bohr's later arguments, his reply assesses EPR's argument on their own, *ontological* terms, whereby it is possible to assign certain properties to quantum objects themselves (under the constraints of the uncertainty relations), rather than in terms of Bohr's later *epistemology*, where, as just explained, such an assignment is no longer possible even for a single physical quantity. Bohr's reply also assumes this assignment to be possible on the basis of predictions rather than only measurements, albeit only predictions that are *in principle verifiable*.

It may appear that EPR's criterion of physical reality applies, *without any further qualification,* in quantum mechanics, just as it does in classical physics. For, only a joint *simultaneous measurement or simultaneous prediction* of two conjugate quantities involved in the quantum-mechanical physical description is impossible in view of the uncertainty relations. The value of a single variable can always be measured with any degree of precision and can be exactly predicted in certain circumstances, such as those of the EPR experiment, which deals with two quantum objects, S_1 and S_2, forming an EPR pair (S_1, S_2), that have previously been in interaction, but are spatially separated. Once S_1 and S_2 are separated, quantum mechanics allows one to establish both the *distance between* the two objects and *the sum of their momenta,* since the corresponding Hilbert-space operators commute. With these quantities in hand, by *measuring* either the position or, conversely, the momentum of S_1, one can *predict exactly* either the position or the momentum for S_2 without physically interfering with S_2 by measurement. EPR thought that these facts enable one to argue for the incompleteness of quantum mechanics. In Bohr's words, 'According to their criterion, the authors [EPR] therefore want to ascribe an element of reality to each of the quantities represented by such variables. Since, moreover, it is a well-known feature of the present formalism of quantum mechanics that it is never possible, in the description of the state of a mechanical system, to attach definite values to both of two canonically conjugate variables, they consequently deem this formalism to be incomplete, and express the belief that a more satisfactory theory can be developed' [2, v. 4, p. 74].

Bohr, as I said, provisionally accepts EPR's criterion of reality in his reply and faults them on the lack of certain qualifications necessary when one applies this criterion to quantum, as opposed to classical, phenomena. If one applies, as EPR do, the criterion without these qualifications, the incompleteness of quantum mechanics would indeed follow. For, given that after the prediction in question, say, concerning the position variable for S_2 is made by virtue of the measurement performed on S_1, one can perform an alternative measurement, that of the momentum variable on S_2. This would allow one to establish the value of *both* variables exactly at the time of this measurement, thus de facto circumventing the uncertainty relations. EPR admit that their argument would not work if one 'insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted*', which *restricts* their criterion of reality [1, p. 141]. In EPR's view, this qualification is unacceptable, since it leads to nonlocality. Once the criterion is assumed to apply in its original form in quantum mechanics, the latter may be assumed to be local but is incomplete. According to this logic, then, quantum mechanics is either incomplete or nonlocal. Bohr contends that neither EPR's argument for the incompleteness of quantum mechanics nor their alternative reasoning leading them to nonlocality is applicable. I shall, now, explain Bohr's reasoning in his critique of EPR's argument for the incompleteness of quantum mechanics, the main target of Bohr's reply. I shall discuss the question of locality in the next section.

One can indeed set up a quantum-mechanical situation, either in the standard or in the EPR case, in two alternative ways so as to predict, by using quantum mechanics, either one or the other of the two conjugate measurable quantities associated with a given quantum object. This assumption may be taken as axiomatic, and it is made by both EPR's argument and Bohr's counterargument. I shall call it '*assumption A*'. From the possibility of the alternative covered by *assumption A*, EPR conclude that *both* quantities can be assigned to the same quantum object, even though it may not be possible to do so simultaneously by means of quantum mechanics, which may be designated as '*inference E*' (for Einstein). Accordingly, quantum mechanics is incomplete (unless one allows for nonlocality), and EPR express a hope that nature should allow for, and some future theory would enable us to offer a more complete description [1, p. 141].

Bohr argues *inference E* to be impossible because of the irreducible role of measuring instruments in any conceivable handling of quantum objects. Given this role, a realization of the two alternative situations of measurement in question, which are necessary for the respective assignment of these quantities (in accordance with the EPR criterion of reality), would, in contrast to classical physics, involve two incompatible experimental arrangements and, thus, *two different quantum objects* of the same type (e.g. electrons or photons). There is no possible physical situation in which this can ever be done for *the same object*, either simultaneously (the uncertainty relations) or separately, or for two 'identically' prepared *objects* in the way it can be in classical physics. I place 'identically' in quotation marks because we can control the identical preparation of the measuring instruments involved, given that this behaviour is classical (i.e. the behaviour of those parts of such instruments that are relevant to measurement, since these instruments also contain quantum strata, through which they interact with quantum objects). One can *never* control the outcomes of thus repeated events, since, in general, identically prepared quantum experiments, including those of the EPR type, lead to different recordings of their outcomes. This fact does not appear to be taken into account by EPR, whose argument in effect depends on the possibility of the identical repetition of the EPR experiment. In the EPR case, one would require a different EPR pair (S_{21}, S_{22}) to get to 'the last critical stage of the measuring procedure', performed on S_{21}, in order to make an EPR prediction of either kind concerning S_{22} [2, v. 4, p. 80]. This inference may be designated as '*inference B*' (for Bohr).

One can diagrammatically represent the situation as follows. Let X and Y be two complementary variables in the Hilbert-space formalism $(XY - YX \neq 0)$ and x and y the corresponding physical measurable quantities $(\Delta x \Delta y \approx h)$; (S_1, S_2) is the EPR pair of the quantum system; and p is the probability of prediction, via the wave function, Ψ. Then:

The EPR experiment (EPR and Einstein's view):

$$\begin{array}{lll} S_1 & & S_2 \\ X_1 & \Psi_1\ (\textit{with}\ p = 1) \rightarrow & X_2 \\ Y_1 & \Psi_2\ (\textit{with}\ p = 1) \rightarrow & Y_2. \end{array}$$

The EPR experiment (the present, Bohrian view):

$$\begin{array}{lll} S_{11} & & S_{12} \\ X_{11} & \Psi_1\ (\textit{with } p = 1) \rightarrow & X_{12} \\ S_{21} & & S_{22} \\ Y_{21} & \Psi_2\ (\textit{with } p = 1) \rightarrow & Y_{22}. \end{array}$$

Bohr himself does not explain the situation in terms of two different objects and EPR pairs necessary in order to make the second EPR prediction, which might have helped his argument. This, however, is at least an implication of his argument, given his insistence in his reply and elsewhere that 'in the problem in question we are not dealing with a *single* specified experimental arrangement, but are *referring* to two different, mutually exclusive arrangements' [2, v. 2, pp. 57, 60; v. 4, p.78]. In view of this mutual exclusivity, due, again, to the irreducible role of the measuring instruments, the second quantity in question cannot in principle be assigned to the *same quantum object* even separately (rather than only simultaneously) in two different experiments, *once one such quantity is assigned.* This is not possible even if one accepts EPR's criterion of reality, whereby such an assignment is made on the basis of a prediction, that is, it is not possible once an experiment enabling one to make the first prediction is performed, since the first object S_1 is no longer available. The simultaneous assignment of both is, again, prevented by the uncertainty relations, which is not only recognized by EPR but is germane to their argument. They see the uncertainty relations as a feature of quantum mechanics and aim to circumvent them by arguing that both variables could in fact *be assigned* to a given quantum object at any point, even though only one of them could be measured or predicted. This is what leads them to reason that quantum mechanics is incomplete, or else nonlocal. Bohr, by contrast, sees the uncertainty relations as a *law of nature, reflected in quantum mechanics*, a law that disallows one to ever assign both quantities in question to the same quantum object.

Nor is it possible, under the conditions defined by *inference B* (Bohr), to coordinate the two experiments in question on two different objects so as to make it possible to consider (in the way it can be done in classical mechanics) both as identically prepared *objects*, or two identically prepared EPR pairs. As noted above, in dealing with quantum phenomena, we can only control our instruments in the same way, but never the behaviour of the quantum objects and, consequently, the outcomes of the experiments defined by this behaviour, which thus become subject to statistical estimates only. In the EPR case, we can predict with the probability unity the first quantity in question, say, the value of the position variable, for the second object, S_{12}, of a given EPR pair (S_{11}, S_{12}). We can then predict the second quantity, the value of the momentum variable, for the second object, S_{22}, of, unavoidably, another, '*identically prepared*', EPR pair (S_{21}, S_{22}). We cannot, however, coordinate these predictions in such a way that they could be considered as pertaining to two identically prepared objects in the way it could be done in classical physics. This is not possible since the necessary intermediate measurements would, in general, give us different data. Were we to repeat the measurement and the prediction of the first pair of quantities, those of the position variables for respectively S_{21} and S_{22}, we could still make our prediction with the probability unity, but the outcome would, in general, not be the same as in the case of the first pair (S_{11}, S_{12}). One *can predict* the outcome of a given EPR experiment with the probability equal to unity but one *cannot repeat* such an experiment with the probability equal to unity for the values of the corresponding outcomes. We can only coordinate such measurements and predictions statistically, and thus establish the EPR correlations (for continuous or discrete variables), as Bell realized. This does not help EPR, since their argument de facto presupposes exact rather than statistical coordination of such variables as belonging to the same, or an *identically prepared*, object of the same, or an identically prepared, EPR pair.

3. Can quantum mechanics be considered local?

As noted above, EPR acknowledge a possible loophole in their argument by admitting that they did not demonstrate that one could ever *simultaneously* ascertain both quantities in question for the same quantum object, such as S_2 above. They, however, see this requirement as implying the nonlocality in the EPR situation and hence as unreasonable. According to EPR:

> One could object to this conclusion [that the quantum-mechanical description of physical reality given by wave functions is incomplete] on the ground that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive to our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted.* On this point of view, since either one or the other, but not both simultaneously, of the quantities P and Q can be predicted, they are not simultaneously real. This makes [in the EPR case] the reality of P or Q depend upon the process of measurement carried out on the first system, which does not disturb the first system in any way. No reasonable definition of reality could be expected to permit this. [2, p. 141]

Nonlocality follows, if one assumes, as EPR do, that the measurement, say, of P, on S_1 *fixes the physical state* of S_2 by 'a spooky action at a distance', rather than allows for 'a spooky *prediction* at a distance', by fixing *the possible conditions* of such a prediction, as explained above. It follows that an alternative measurement of y on S_2 would discontinuously change this fixed state, although EPR do not examine this last eventuality. Or, as Einstein argued later, one is left with a paradoxical situation insofar as (assuming that quantum mechanics is complete) two mutually incompatible states could be assigned to the same quantum system [5, 6]. This is why EPR argue that, if quantum mechanics *is* complete by their criterion of *completeness*, then the physical state of a system, here S_2, could be *determined* by a measurement on a spatially separate system, S_1, in violation of locality, while their criterion of *reality* no longer applies in its original form. If it is local, their main argument, based on their criterion of reality, shows (they believe) that it is incomplete. Einstein thought that Bohr accepted the alternative of locality versus completeness, and retained completeness by allowing for nonlocality [5, p. 681]. Einstein, however, misread Bohr, who, again, only allows for a spooky *prediction*, and *not action*, at a distance.

According to Bohr, 'the singular position of measuring instruments in the account of quantum phenomena ... together with the relativistic invariance of the uncertainty relations... ensures the compatibility between [the] argument [of his reply] and all exigencies of relativity theory' [2, v. 4, p. 82n., also v. 3, p. 3]. This compatibility enables Bohr's critique of EPR's argument for the incompleteness of quantum mechanics, without sacrificing locality.

There is a subtle difference between determining the state *by a* prediction and doing so *on the basis* of a prediction. In Bohr's view physical states cannot be seen as finally determined (even when we have predicted them exactly) unless either the actual measurement is made or the possibility of *verifying* the prediction is assured insofar as such a measurement could, in principle, still be performed so as to yield the predicted value. This condition in turn becomes a necessary qualification of EPR's criterion of reality in the case of quantum phenomena. For, if one assumes the validity of EPR's criterion in its original (unrestricted) form, the measurement of the alternative quantity, Q, on S_2 would automatically disable any possible verification of the original prediction. Once this measurement is performed, the original prediction becomes meaningless as in principle unverifiable. This, again, implies that both quantities in question could never be experimentally ascertained, either simultaneously or separately, for the same quantum object and hence that quantum mechanics could not be shown to be nonlocal by this logic, anymore than it can be shown to be incomplete by their logic, as discussed above.

This is why Bohr argues, with EPR's alternative between locality and completeness in mind, that it is not a question of physical or 'mechanical' influence of the measurement on S_1 upon the physical state of affairs concerning S_2. As he says: 'of course there is in [the EPR] case no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure' [2, v. 4, p. 80]. The 'influence' here should not be seen as an influence, *at a distance*, of the measurement performed on S_1 upon the spatially separated situation of S_2, although this measurement defines our predictions concerning the corresponding future measurement on S_2. Bohr makes this point clear before the statement just cited: 'there is essentially the *question of an influence on the very conditions which define the possible types of predictions, regarding the future behaviour of the system* [S_2]' [2, v. 4, p. 80]. The influence in question is defined by *fixing* the conditions in the EPR situation by the particular measurement on S_1 as mutually exclusive for each physical quantity in question, which, accordingly, strictly determines the kind of quantity that can be predicted for S_2. This influence concerns the conditions of the measurement on S_1 and, correspondingly, the prediction, the only possible prediction by virtue of this measurement, concerning S_2. It concerns the determination of one possible experimental set-up, as opposed to the other possible set-up, and it is never possible to combine both set-ups. Once one of the two possible set-ups is in place and defines the measurement on the first object, any determination of the second quantity becomes impossible. An alternative arrangement, which would make this type of determination possible, would inevitably involve a different quantum object. Our decision *influences* what kind of predictions concerning S_2 become possible, even though we do not interfere with it, while Bohr clearly takes the requirements of relativity and, hence, locality as axiomatic, just as Einstein does.

4. Conclusion

EPR's argument is disabled by the nature of quantum phenomena themselves, as defined by the irreducible role of measuring instruments in the constitution of these phenomena, and hence by the impossibility of considering the behaviour of quantum objects independently of their interaction with these instruments. The application of EPR's criterion of reality becomes ambiguous by virtue of the lack of the qualifications of this criterion required by these conditions. 'Since', Bohr concludes, 'these conditions constitute an inherent element of the description of any [quantum] phenomenon to which the term 'physical reality' can be properly attached, we see that the argument of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete' [2, v. 4, p. 80]. Instead these conditions entail 'the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude toward the problem of physical reality', while allowing for the locality of quantum phenomena and quantum mechanics [2, v. 4, p. 75, p. 82n.].

'Subtle is the Lord, but malicious he is not', Einstein famously said. He might have been more right in his view of the Lord or nature than in his view of quantum mechanics. The Lord or nature and quantum mechanics alike may be subtler than he thought or subtle in a different way.

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