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Experimenting with ontologies: sets, spaces, and topoi with Badiou and Grothendieck

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Abstract. The paper explores the ontology and logic of the irreducibly multiple in set theory and in topos theory by considering the differences between Badiou's logical and Grothendieck's ontological approach to topos theory. It argues that Grothendieck's ontological program for topos theory leads to a more radical concept of the multiple than does the set-theoretical ontology, which defines Badiou's view of ontology even in his later, more topos theoretically oriented work. Extending Grothendieck's way of thinking to other fields enables one to give ontological multiplicities—no longer bound by the set-theoretical ontology or ultimately by any mathematical ontology, even in mathematics—a great diversity and richness. It follows that the set-theoretical ontology is not sufficiently rich to accomplish what Badiou thinks it could accomplish even in mathematics itself, let alone elsewhere; and Badiou wants it to work elsewhere—indeed, wherever it is possible to speak of ontology. I shall also consider, in closing, some implications of the arguments for the workings of the multiple in ethics, politics, and culture.

Keywords: logic, mathematics, multiplicity, ontology, philosophy, set theory, topos theory

1 Introduction

The mathematical grounding of Alain Badiou's philosophy is arguably the most distinctive aspect of his work; it also distinguishes it from the work of other contemporary authors who have impacted recent theoretical discussions. It is true that both Jacques Lacan and Gilles Deleuze notably engage with mathematics, and they do offer some competition to Badiou in this regard. Badiou's use of mathematics is, however, more extensive and more foundational in view of the grounding roles of mathematical ontology and mathematical logic in his work. Badiou (2007, page 8) even contends that a rigorous philosophical ontology can only be mathematical, at least as "a thesis ... about discourse" ("mathematics ... pronounces what is expressible about being qua being"). Badiou (2006, pages 166 – 168) holds a similar view on rigorous philosophical logic: it could only be, or in any event should be, mathematical.[1]

Two mathematical theories are especially important for Badiou: set theory, in establishing ontology, and category theory in establishing logic (or more specifically, topos theory, as grounded in category theory). Badiou argues that set and category theories are now the only ones that provide "the apparatuses that claim to endow mathematics with its own unified space, or primordial languages" (2006, page 105). Set theory, introduced by Georg Cantor in the late 19th century, grounds Badiou's earlier work, in particular *Being and Event* (2007 [1988]). This book develops a form of set-theoretical ontology, which is defined by its irreducibly multiple character—that is, by the impossibility of finding a unity within which the multiple of this ontology could in principle be subsumed. Badiou speaks of this ontology as that of "the multiple-without-One" (eg, page 29; Badiou, 2006, page 35). Badiou's later

[1] I shall refer extensively to this work *Briefings on Existence*, because, in addition to a useful set of briefings on Badiou's thought, it provides Badiou's arguably best articulation of his use of topos theory.

work, from the early 1990s on, is marked by its engagement with category theory, which was introduced by Samuel Eilenberg and Saunders Mac Lane in the 1940s, and especially by its engagement with topos theory, which was introduced by Alexandre Grothendieck in the 1960s (and which was based on category theory). In contrast to set theory, topos theory is used by Badiou as (mathematized) logic rather than ontology, although the theory was developed by Grothendieck along ontological lines, indeed as a form of mathematical ontology (although Grothendieck himself did not use ontological language in commenting on topos theory). Shortly after its introduction, however, topos theory was used in mathematical logic in the work of Peter G Freyd, Francis W Lawvere, and others, which grounds Badiou's use of the theory.

The main aim of this paper is to explore the ontology and logic of the irreducibly multiple in set theory and in topos theory, in part by considering the differences between Badiou's logical and Grothendieck's ontological approach to topos theory. I am in agreement with Badiou's (2006) argument that "mathematics is a thought" (page 45) and often an ontological thought rather than merely a form of logic. Indeed, given this argument, I find it somewhat peculiar that Badiou is willing, in effect, to relinquish this view when it comes to the mathematics of topos theory and to see the latter as logic, albeit a properly mathematized logic. In this respect the present paper aims to restore topos theory to mathematics as a thought and to reconnect it with ontology, without relinquishing the logical dimensions and potential of the theory. Further, I argue that Grothendieck's ontological program for topos theory leads to a more radical concept of the multiple than does the set-theoretical ontology, which defines Badiou's view of ontology again, even in his later, more topos-theoretically oriented work. Thus, by virtue of its strictly set-theoretical character, Badiou's ontology is ultimately that of *one* multiple-without-One, possibly with a different form of logic provided by topos theory governing this ontology in, in Badiou's terms, each "situation" or (in later works) "world". Badiou does not appear to entertain the possibility of ontologies that are other than set-theoretical, let alone other than mathematical. By contrast Grothendieck's topos-theoretical ontology is that of the *multiple* of multiples-without-Ones: it is a multiplicity of possible ontologies, each of which may be governed by multiple possible logics. Extending this type of thinking to other fields enables one to give the corresponding multiples of multiples a great diversity and richness, which is no longer bound not only by the set-theoretical ontology but also by any mathematical ontology, however multiple. Accordingly, contrary to Badiou's view that any ontology rigorously considered by philosophy should be mathematical, I shall argue that ontology—that which "pronounces what is expressible about being qua being"—need not always be and even cannot always be mathematical, or only mathematical. Indeed, I shall argue that this is true even in mathematics itself: no matter how we try to configure it mathematically (via set theory, topos theory, or otherwise), any rigorously established mathematical ontology always has a nonmathematical residue. However, an ontology exceeding mathematics, in mathematics itself or elsewhere, can be established in rigorously philosophical terms. This also tells us again that, contrary to Badiou's view, we need not, and may not be able to, think of philosophical rigor only in terms of mathematical (fully "exact") rigor. In sum, I shall argue that the set-theoretical ontology may not be sufficiently rich to accomplish what Badiou thinks it could accomplish even in mathematics itself, let alone elsewhere and Badiou wants it to work elsewhere, indeed, wherever it is possible to speak of ontology. To support this argument, I shall, in closing, also address some implications of this paper's argument for the workings of the multiple in ethics, politics, and culture.

2 From sets to topoi: mathematics, logic, and philosophy in Badiou

According to Badiou, then, "mathematics is a thought", a thought concerning being as a-multiple-without-One (2006, pages 45 – 62). This means, in particular, that the mathematical being qua being is brought into existence by the movement and decision of thought, "wherein discovery and invention are strictly indiscernible" (page 94). Although mathematical thinking embodies consistency (also when thinking about the inconsistent), it is not merely a logical game. It is a practice defined by decisions of thought concerning what actually exists and is true for thought.

By contrast, although equally "about identifying what real ontology is", philosophy exceeds (mathematical) ontology, especially as the transontological thinking of "event" (a crucial concept for Badiou), since an event is always an event of "trans-Being". As will be seen, however, philosophy does not exceed mathematics itself, which as topos theory (seen as mathematized logic), ultimately defines the transontological thinking of the event (page 59). Both "event" and "truth" are defined by Badiou in relation to what he calls "situation" or in *Logics of Worlds* (2009 [2006]), "world": that from within which, but also by breaking with which, an event emerges. Indeed, one might see Badiou's philosophical project as primarily motivated by his concern with the eruptive and thus revolutionary nature of events, wherever they occur, rather than by his concern with the mathematical-like rigor of philosophical thinking. This latter concern, however, remains crucial for his philosophy, especially insofar as concerns the grounding of his project through his view of both ontology (via set theory) and the logic of "event" (via topos theory). The eruptive emergence of an event (always a crisis) and thus the ultimate nature of thought are beyond ontology, albeit, again, not beyond mathematics.

Mathematics, too, can think philosophically, especially at times of "crises" and on its way to "events". As Badiou (2006) says, "a 'crisis' in mathematics arises when it is compelled to think its thought *as the immanent multiplicity of its own unity*", and "it is at this point, and only at this point, that mathematics—that is, ontology—functions as a condition of philosophy. Let us put it in this way: mathematics relates to its own thought according to its orientation. It is up to philosophy to pursue this gesture by way of a general theory of orientation of thought" (page 54). In other words, the ontology may be the same or be of the same type—indeed, it is always the set-theoretical ontology. By contrast, although within the same or isomorphic ontological architecture, the *orientation* of thought in mathematics and beyond may be different (pages 55 – 57). Accordingly, so too may be the logic that shapes this orientation (pages 166 – 168). Philosophy is thinking this difference. Mathematics, however, reenters the scene of philosophical thought, as understood by Badiou, via this logic, defined as topos-theoretical mathematical logic.

Accordingly, in order to understand how the *multiplicity of ontology* and the *discontinuity of event* work together in Badiou (2006) and define the reciprocity and yet difference between mathematics and philosophy, it is necessary to first consider set theory as opening "the very space of the *mathematically thinkable*" and then topos theory. The latter extends this opening to a fundamental "*geometrization* of the relation and 'de-relation' between logic and mathematics", and connects being and appearance and being and event (pages 42, 113, 166 – 168, emphasis added). Hence, as I said, while the emergence of an event exceeds mathematics as ontology, it must still be ultimately thought of mathematically, by way of rigorously mathematized logic as topos theory, logic made topos-theoretical mathematics. The remainder of this section is devoted to the role of set theory in Badiou. I discuss topos theory in Grothendieck and Badiou in the next section.

I shall, for the most part, remain within the limits of the so-called "naïve" set theory, as opposed to more rigorous versions of it (eg, axiomatic, constructible, or generic), which deal with the complexities and paradoxes of the theory, and which Badiou uses in his argumentation. A set is a collection of objects, called elements of the set, for example, numbers between 1 and 10, which is a finite set, or of all natural numbers (1, 2, 3, 4, etc), which is an infinite set, a *countable* infinite set as it is called. There are greater infinities, such as that of the continuum, which intuitively corresponds to the numbers of points in a straight line. The resulting ontological multiplicity (that of all sets) is, as Badiou stresses, unavailable to unification, to the One, and this multiplicity is also inconsistent. These two features are crucial to the set-theoretical ontology pursued by Badiou. Our thought itself concerning this inconsistent multiplicity must be consistent (it could not be mathematically rigorous otherwise). While with Cantor we recognize "not only the existence of infinite sets, but also the existence of infinitely many such sets" (sets possessing different magnitudes of infinity); "this infinity itself is absolutely open ended" (Badiou, 2006, page 41, translation modified). In particular, this infinity cannot be contained in a set. There is no "the One" of set theory because *the set of all sets does not exist*. Or at least such a set cannot be consistently defined in view of the well-known paradox related to the question of whether the set of *all* sets that do not contain themselves as their elements, does or does not contain itself as an element. It is easily shown that such a set cannot be consistently defined, and it follows that a set of all sets cannot be consistently defined either. According to Badiou, this "multiple is radically without-One in that it itself consists only of multiples. What there is, or the exposure to the thinkable of what there is under the sole requirement of the 'there is,' are multiples of multiples ... . Ontology is the thought of the inconsistent multiplicity" (pages 35–41, translation modified).

While this ontology can be established on the basis of the more general set-theoretical considerations just outlined, its deeper complexities are revealed by Kurt Gödel's famous discovery of the existence of undecidable propositions in mathematics in 1931 and then by Paul Cohen's findings concerning Cantor's continuum hypothesis in 1963. An undecidable proposition is a proposition the truth or falsity of which cannot be established by means of the system (defined by consistent axioms and rules of procedure) in which it is formulated. Gödel's discovery undermined the thinking of the whole preceding history of mathematics, defined by the assumption that every mathematical proposition can, in principle, be shown to be either true or false. Gödel proved—rigorously, *mathematically*—that any system sufficiently rich enough to contain arithmetic (otherwise the theorem is not true) would unavoidably contain at least one undecidable proposition. This is Gödel's "first incompleteness theorem". Gödel made our foundational thinking in mathematics even more difficult with his "second incompleteness theorem", by proving that the proposition that such a system, say classical arithmetic, is consistent, is itself an undecidable proposition. It follows that the consistency of most of the mathematics we use cannot be proven, although the possibility that this mathematics may be shown to be inconsistent remains open.

In set theory the role of undecidability becomes especially dramatic in view of Cantor's continuum hypothesis. It states, roughly, that there is no infinity larger than that of a countable set (such as that of natural numbers) and smaller than that of the continuum. The latter, as I said, intuitively corresponds to the number of points on the straight line and is defined set-theoretically as the set of all subsets of the set of natural numbers. The hypothesis is crucial if one wants to maintain Cantor's hierarchical order of infinities and hence for the whole edifice of set theory. However, the hypothesis was proven undecidable by Cohen in 1963. It follows that one can extend classical arithmetic in two ways by considering Cantor's hypothesis as either true or false—that is, by assuming either that there is no such intermediate infinity or that there is.

This allows one, by decisions of thought, to extend arithmetic into two mutually incompatible systems and (by iteration) to infinitely many such systems—an intolerable situation for some. Badiou (2006), by contrast, finds in it a special appeal, and as will be seen, a model of politics:

"As we have known since Paul Cohen's theorem, the Continuum Hypothesis is intrinsically undecidable. Many believe Cohen's discovery has driven the set-theoretic project into ruin. Or at least it has "pluralized" what was once presented as a unified construct ... . [M]y point of view on this matter is ... the opposite. What the undecidability of the Continuum Hypothesis does is complete Set Theory as a Platonist orientation. It indicates its line of flight, the aporia of immanent wandering in which thought experiences itself as an unfounded confrontation with the undecidable. Or, to use Gödel's lexicon: as a continuous recourse to intuition, that is, to decision" (page 99).

The situation just outlined defines Badiou's ontology from *Being and Event* to *Logics of Worlds*. However, as noted above, it does not define the nature of "event". To do so, within Badiou's scheme, requires topos theory, but in combination with the set-theoretical ontology of the multiple, in contrast to Grothendieck's approach, which is no longer anchored in this ontology.

3 Topology, ontology, and logic: topos theory in Grothendieck and Badiou

Topos theory is almost prohibitively difficult in view of its highly abstract and technical nature, and there is virtually no sufficiently nontechnical literature that would make it accessible for the lay reader. The essential philosophical ideas behind it are, however, possible to convey, which I shall attempt to do now. I shall speak primarily of spatial objects for the sake of intuitive convenience and because one of my aims is to explore the topos-theoretical-like conceptual architecture of spatiality in and beyond mathematics. Topological considerations were also crucial for Grothendieck's ontological approach, in contrast to the logical version of topos theory. In Grothendieck's topos theory a space-*like* concept of topos is the primary concept, in particular, more primary than that of the set. When applied to spatiality proper, the theory allows one to make 'space' a more primary concept and object than 'point' or 'set of points', which concept defines the set-theoretical view of spatiality that has dominated mathematics since Cantor. However, the *concept* of topos is ultimately algebraic. Topos theory adopts certain algebraic properties pertaining to topological spaces, particularly those defined by the relationships among them, and between them and certain algebraic objects, especially the so-called 'groups', and generalizes these properties to objects other than spatial ones. This places certain restrictions upon those multiplicities ('categories' in the mathematical sense of the term, explained below) that form topoi. What topological spaces in the usual sense and topoi share is their architecture. This commonality also leads one, in dealing with algebraic objects amenable to this type of treatment, to speak (more metaphorically) of a 'geometrization' of algebra, although 'topologization' would be more accurate. One can also see topos as a new, *algebraic* concept of space, which applies to conventional spatial objects but extends beyond them.

Historically, Grothendieck's work followed the use of algebra in topology, which defines topology as a mathematical discipline separate from geometry. While both geometry and topology are concerned with space, they are distinguished by their different ways of studying space. Geometry (geo-*metry*) has to do with measurement; topology disregards measurement and scale and deals only with the structure of space qua space, for example, with the *essential* shapes of figures, seen as continuous spaces. Distances are generally irrelevant. It is only continuity (as connectivity) or conversely rupture of continuity that matters, which is why topology defines space via its so-called open subspaces, such as those (called neighborhoods) around each point. A good model of an open subspace in the two-dimensional case is a circle considered without its circumference. On a two-dimensional surface one can think of a neighborhood of a given point as a small circle (without boundaries) around this point. Insofar as one deforms a given figure continuously (ie, insofar as one does not separate points previously connected and, conversely, does not connect points previously separated), the resulting figure is considered the same. The proper mathematical term is 'topological equivalence'. Thus, all spherical surfaces, of whatever size and however deformed, are topologically equivalent, although some of the resulting objects are no longer spherical, geometrically speaking. Such figures are, however, topologically distinct from the surfaces of tori, since these two kinds of surfaces cannot be transformed into each other without disjoining their connected points or joining the disconnected ones: the holes in tori make this impossible. This is sometimes expressed by saying that, rather than measuring distances, as with geometry, topology 'measures' (counts) the number of holes in a spatial object.

Topology is mathematical not by virtue of mathematizing spatiality by measurement, as geometry does, but by virtue of relating the architecture of spatial objects to algebraic or numerical properties of algebraic or arithmetical objects. The number of holes in a given object—such as the surface of a sphere with no holes in it vis-à-vis that of a torus, which has one hole in it (or the surfaces of pretzel-like figures, each with several holes in it)—is the simplest example of this kind of relation. The field of topology known as 'algebraic topology' studies topological spaces by relating them to algebraic objects, particularly the so-called groups, defined by abstract elements and a multiplication-like operation upon them, resulting in the elements of the same kind. Thus (glossing over technical specifics), whole numbers form a group with respect to addition, but not multiplication, since an inverse of a whole number is a fraction. Rotations of the circle or of the two-dimensional surface of the three-dimensional sphere also form a group, with its operation defined as that of performing consecutive rotations. In the case of the surface of the sphere, the order of rotations may change the outcome, which is expressed mathematically by saying that this group is not commutative. Groups play a major role in algebraic topology, and in category and topos theories, especially in the so-called homotopy and cohomology theories, which deal with certain groups, defined by the topology of the corresponding spaces and essential for studying these spaces. In the case of two-dimensional surfaces these groups are linked to the numbers of holes in them, as described above, and these groups are, accordingly, different for spheres and tori, for example.

Category theory, which grounds topos theory, was initially developed by Eilenberg and Mac Lane as part of algebraic topology, specifically in order to define cohomology groups for certain spaces, for which previous methods of defining such groups had failed, which made it difficult to study the topological architecture of these spaces. Roughly, a category is a multiplicity (it need not be a set) of mathematical objects (which need not be spaces) conforming to a given concept, such as the category of topological spaces, and the 'morphisms', which are those mappings between these objects that preserve the structures defined by this concept. Morphisms are sometimes called 'arrows', because of the corresponding notation, $Y \Rightarrow X$, designating a morphism between X and Y. There are additional rules concerning morphisms (eg, they form a group). Categories themselves may be viewed as such objects, and in this case one speaks of 'functors' rather than 'morphisms', and the functors can form categories, too, with their corresponding morphisms: the process is, in principle, infinite. The point here, however, is not this interminable categorical build-up for its own sake (although the 'game' is played and has its purchase, for example, in computer science). Instead, the process can be used to reach the level necessary for a particular task, such as in Grothendieck's topos theory, creating an extension of the concept of topology and, correlatively, cohomology theory to new types of objects or 'spaces'.

From categorical or topos-theoretical perspective one starts with a certain, arbitrarily chosen space, X, potentially any space, without initially specifying it mathematically. Indeed, one can start with an object, say, a set of numbers, that is not spatial in any given sense and only becomes spatialized by virtue of its relations to other objects of the same kind, analogous to the relationships between conventional spatial objects. What would be specified, at first, are the relationships between spaces, such as categorical arrows, $Y \Rightarrow X$, mapping one space by another space or multiplicity of other spaces. This procedure enables one to specify a given space not in terms of its intrinsic structure, say, as a set of points with relationships among them, but, in Yuri I Manin's terms, 'sociologically'—through its relationships with other spaces of the same category (Manin, 2002, page 7). An intrinsic structure—set-theoretical or other, say, topological, as the number of holes in a given space—is then derived from this 'sociology'.

Topos theory was a culmination of Grothendieck's program for algebraic *geometry*, which is different from algebraic *topology*. Algebraic geometry emerged in the mid-19th century as an extension of Descartes's analytic geometry to abstract spaces, algebraic varieties, defined by algebraic (polynomial) equations with any number of variables, rather than at most three, as in Descartes. Some algebraic varieties are regular topological spaces, say, two-dimensional surfaces, such as those of spheres or tori, and these algebraic varieties can, accordingly, be automatically treated by means of the standard cohomological methods of algebraic *topology* (again, not the same as algebraic geometry!). But some algebraic varieties (solutions of certain algebraic equations) are not *automatically* amenable to a treatment by these methods, in the first place, because unlike, say, a standard two-dimensional spherical surface, these varieties are essentially discrete if one considers them as standard topological spaces. Without, however, having in place a machinery of the type algebraic topology offers, it would be very difficult to understand and study the structure of these algebraic varieties. This is important for many mathematical tasks—for example, for a proof of Fermat's last theorem by Andrew Wyles, one of the great achievements of contemporary mathematics.

Grothendieck's project aimed to construct the workable machinery, technology, analogous to that of algebraic topology in this domain. The possibility of developing this type of mathematical technology was conjectured earlier by André Weil as part of the so-called Weil's conjectures. Grothendieck, however, realized that the task could be accomplished via category theory, a theory, again, initially developed for similar purposes in algebraic topology. As noted above, in contrast to geometry (which relates spaces to algebraic and numerical features by measurement), topology by its very nature deals with functors between categories of topological objects, spaces, and categories of algebraic objects, particularly groups, as in cohomology theory. Accordingly, two tasks needed to be accomplished for the algebraic varieties in question. First, one needed to construct a nonstandard topology in which these varieties would become more "continuous" spaces, which means that they would have more subspaces; in other words, one needed to enrich them topologically. Second, one needed to construct proper functors from the category of algebraic varieties to the category of groups, functors analogous to those defining the cohomology groups of the standard algebraic topology (which is the essential content of Weil's conjectures). This program essentially amounts to the enrichment of the mathematical architecture associated with the algebraic varieties in question. Grothendieck and his coworkers were able to achieve this by means of topos theory via several intermediate steps, which are not essential here.

A topos is a category of spaces (with arrows) over a given space. There are certain additional conditions that *Grothendieck's topoi* (rather than topoi used in mathematical logic) must satisfy as categories. These conditions have to do with the fact that some of the defining properties of topoi are, as indicated above, parallel to certain algebraic properties associated topological or algebraic – geometrical spaces and their categories, or algebraic categories related to them. Thus, these are the same conditions that make topos a new ('sociologically') algebraic concept of spatiality. The concept enables one to endow such objects with a richer topology, defined by the sociology of spaces and their relations and again, correlativity, to associate a richer algebraic architecture with them, even for spaces consisting of a single point. A point obviously has no subspaces, but it can be related to or, as it were, covered by other spaces, which form a topos that 'sociologically' endows this point with a rich mathematical architecture. There are also topoi without points, which concept prompted some mathematicians to slyly refer to topos theory as a pointless topology, a joke with a grain of truth, given the overly abstract nature of the theory, which makes some shun it. But, it is also the semi-official name of a subfield, in which the primacy of space over point is emphasized. The working slogan is 'points come later', that is, after the architecture of space is sociologically defined via a topos of other spaces over a given space. From this viewpoint, rather than a given point's point-like nature according to the conventional topology (which makes all points the same), it is the mathematical architecture that can be associated with a given point that defines it, which no longer makes all points the same. It is not unlike the situation in geography when the meaning of a physically spatial point depends on what kind of map (physical, economic, political, or other) or, closer to a topos, atlas of maps is associated with it.

As this discussion makes apparent, Grothendieck's topos-theoretical thinking is primarily ontological. Grothendieck's theory replaces or complements the set-theoretical ontology of the objects considered with a more multiple ontology. For Grothendieck a topos is a *topos*, an algebraic concept of spatial type, rather than *logos*, in the sense of the topos-theoretical rendering of mathematical logic, as it is for Lawvere and other logicians and following them, Badiou. Badiou sees topos theory as a theory explaining "the plurality of possible *logics*", rather than as an ontology like the one set theory provides for him or the plurality of possible ontologies, as in Grothendieck (*BE* 2006, page 166, emphasis added). The theory offered a rich field of possibilities for the field of mathematical logic in view of the surprising and remarkable fact that one can map various forms of mathematical logic, classical (that of the excluded middle), intuitionist, or other, by the corresponding topoi. Given, however, the manifest ontological potential of Grothendieck's theory and Badiou's ontological interests, one might ask: Why only logic or even primarily logic? Why did Badiou, for whom mathematics' primary significance is in offering ontology, choose to bypass this potential? The reasons are subtle, and I shall now explain them. According to Badiou:

> "The theory of *toposes* is descriptive and not really axiomatic. The classical axioms of Set Theory lay out an untotalizable universe of the thought of pure multiple. Say that Set Theory is an ontological decision. *Topos* theory defines the conditions beneath which it is acceptable to speak of the universe of thought, based on the absolutely impoverished concept of relation-in-general. Consequently, we may also speak of the localization of a situation of Being. To spin a Leibnizian metaphor: Set Theory creates 'through fulguration', a singular universe in which what 'there is' is thought according to its pure 'there is.' *Topos* theory describes the possible universes and their rules of possibility. It is like inspecting the possible universes

Leibniz conceived of as in God's Understanding, as it were. This is why it is not a mathematics of Being but mathematical logic.

Topos theory explains the plurality of possible logics. This point is crucial. Indeed, if Being's local appearing is intransitive to its being, there is no reason why logic, which is the thought of appearing, should be unique. The linkage form of appearing, which is the manifestation of the 'there' of Being-there, is itself a [multiplicity]. *Topos* theory allows us to understand in depth from the mathematicity of possible universes where and how logical variability, which is the contingent variability of appearing as well, is marked in relation to the strict and necessary univocity of Being-multiple. For example, there can be classical *toposes* that validate internally the excluded middle or the equivalence of the double negation with affirmation. There can be nonclassical logics, as well, which do not validate either of these two principles" (*BE* 2006, pages 166 – 167, also page 119, translation slightly modified).

It is clear that this assessment applies only to the logical but not to Grothendieck's ontological version of topos theory. The main reason for Badiou's use of topos theory as logic is the necessity of rethinking "the gap between logic and mathematics" (page 119). This gap appears and is a problem if one thinks mathematics, as Badiou does, along the lines of the Platonist orientation, adjusted to the ontology of the-multiple-without-One (page 95), as against the Aristotelian orientation. The former conceives of mathematics as a thought and as ontology of thought and as *axiomatic* and not descriptive. The latter is, broadly speaking, linguistic. As such, it in effect denies thought to mathematics and makes it a grammar or logic of the possible (page 102). Most of the modern mathematical logic and philosophy of mathematics, from Frege and Russell on, have followed this Aristotelian orientation, although there are notable Platonist exceptions, such as Gödel (page 92). According to Badiou, the Aristotelian orientation in mathematical logic is not really mathematical, since it does not deal with mathematics as a thought but merely sees it as a formal game following a given set of rules. In this orientation, "mathematics is ... ultimately a rigorous aesthetics. It tells us nothing of real-being, but it forges a fiction of intelligible consistency from the standpoint of the latter, whose rules are explicit" (page 48). In other words, in this view mathematics is a science of appearances without a proper relation to being and truth, a relation that Badiou wants to establish by rethinking, first, mathematics as a thought via set theory and second, logic via topos theory.

This rethinking of logic is necessary because of the following problem. As a thought and ontology, mathematics is more than logic alone and is indeed profoundly different from logic. But it cannot do without logic. In particular, as explained above, while, the set-theoretical ontology as the ontology of the multiple-without-One, deals with an inconsistent multiplicity, mathematical procedures of dealing with it, and hence, our ontological thought, must be logically consistent. The Platonist orientation does "give precedence to *decided* [axiomatic] *consistencies* over controlled constructions", but it is as consistent as the Aristotelian orientation" (page 96). One needs, accordingly, to liberate mathematical logic from its Aristotelian orientation and to unite it with mathematics as a thought, to the rigor of logic to the rigor of ontology, and hence to the truth of Being, and ultimately use logic to reach the truth of the event as trans-Being.

The *logical* topos theory, mapping a manifold of logics of possible mathematical worlds, responds to this task: it properly connects mathematics, as a thought, and logic, and also, and correlatively, properly relates being and appearing. As we have seen, according to Badiou, "the purely logical operators are not presented in a *topos* as linguistic forms. They are constituents of the universe that are in no way formally distinguished from other constituents. ... Truth itself is but an arrow of the *topos*, the truth-morphism. As such logic is nothing else than a particular power of localization immanent to such or such a possible universe" (page 167). Thus, there emerges a *unity*, but not *identity*, of logic and mathematics, and of logic and ontology within a single scheme, arising from the strict *mathematicity* of logic as topos theory. For "*topos* theory allows us to understand in depth from the mathematicity of possible universes where and how logical variability, which is the contingent variability of appearing as well, is marked in relation to the strict and necessary univocity of Being-multiple" (page 167). "For these reasons", Badiou concludes, "we can assert that [topos] theory to be indeed mathematical logic as such" (page 167). By the same token, topos theory may also be seen as a kind of mathematical phenomenology, for "within ontology, it is the science of appearing, that is, the science of what signifies that every truth of Being is irremediably a local truth. What we can read in it ... is that the science of appearing is also and at the same time the science of Being *qua* Being, in the specific inflection inflected upon it by the place earmarking a truth for it" (page 167). It is this unity of being and appearing that according to Badiou should ultimately enable us to understand the nature of what he calls an event as trans-Being. As explained earlier, Badiou's conception of "the event as trans-Being" establishes the difference between mathematics as ontology and philosophy as that which, while concerned with "identifying what real ontology is", is ultimately released from ontology. As Badiou writes:

> "A vast question opens up regarding what is subtracted from ontological determination. This is the question of confronting what is not Being *qua* Being. For the subtractive law is implacable: if real ontology is set up as [mathematical ontology] by evading the norm of the One, unless this norm is reestablished globally there also ought to be a point wherein the ontological, hence mathematical, field is de-totalized or remains at a dead end. I have named this point the '*event*'. While philosophy is all about identifying what real ontology is in an endlessly reviewed process, it is also the general theory of the event—and it is no doubt the special theory, too. In other words, it is the theory of what is subtracted from ontological subtraction. Philosophy is the theory of what is strictly impossible for mathematics" (page 60).

In this task topos theory (as logic) becomes, according to Badiou, indispensable. As he says: "What draws philosophy—under the condition of mathematics—to rethink Being according to what is, in my view, a contemporary program, is the task of understanding how it is possible for a situation of nondescript being to be both a pure [multiple] at the selvage of inconsistency, and an intrinsic and solid linkage of its appearing" (page 168). This program is taken on in the more Hegelian *Logic of the World* (originally published in French in 2006), which offers a kind of phenomenology to the ontology of the multiple-without-One introduced in *Being and Event* (originally published in French in 1988). It is, Badiou argues, only once this task is accomplished, that understanding the nature of event, in which Being transforms its logic and orientation, could be possible. As he says:

> "Only then shall we know why, when a novelty is shown, when the Being beneath our eyes seems to shift its configuration, that this always occurs for want of appearance—in a local collapse of Being's consistency and thus in a provisional termination of any logic. For what then surfaces, what displaces and revokes logic from the place, is Being itself in its redoubtable and creative inconsistency. It is Being in its void, which is the non-place of every place" (2006, page 168).

An 'event' is an occurrence in which a given 'situation' and its logic, its being and its appearing collapse, and a new situation and new logic must emerge. Through its topos-theoretical relation to appearing, Being comprises, from its two different sides, the multiple-without-One and the void. As Badiou writes:

"This is what I call an 'event'. All in all, it lies for thought at the inner juncture of mathematics [as ontology] and mathematical logic. The event occurs when the logic of appearing is no longer apt to localize the manifold-being of which it is in possession. As Mallarmé would say, at that point one is then in the waters of the wave in which reality as a whole dissolves. Yet one also finds oneself where there is a chance for something to emerge, as far away as where a place might fuse with the beyond, that is, in the advent of another logical place, one both bright and cold, a Constellation" (page 168).

The ability to think the place or space of an event certainly offers a strong justification for Badiou's use of topos theory as logic. Indeed, it is intriguing, but given Badiou's insistence on identifying rigor with mathematical rigor, not surprising, that this excess of both mathematical ontological thought in philosophy's thinking of an event is achieved by means of thought that is still mathematical—the strictly mathematized topos-theoretical logic. Nevertheless, it is still difficult not to question Badiou's decision (a decision of thought?) to place topos theory in a strictly descriptive logical register, à la Lawvere, as against Grothendieck's ontological program, aimed at a plurality of possible ontologies rather than only of possible logics, or the plurality of both. This decision poses certain important and pressing questions even from within Badiou's own argument, questions not addressed by Badiou. Thus, if "Set Theory creates 'through fulguration,' a singular universe in which what 'there is' is thought according to its pure 'there is,' while "*topos* theory describes the possible universes and their rules of possibility," does this mean that one needs a set-theoretical ontology to decide upon which universe is real (page 166)? Or, given that not all topoi are those of sets, are there forms of mathematical ontology of the multiple-without-One that are not set-theoretical? And would not different set-theoretical logics imply that different set-theoretical ontologies are possible (page 56)?

Badiou is unlikely to be entirely unaware of Grothendieck's theory's ontological potential or ambition, as *Logics of Worlds* (2009 [2006]) would indicate. This is a later work (by a decade) than *Briefings on Existence* [2006 [1996]), primarily discussed thus far, and it might be developing Badiou's earlier logical thinking concerning topos theory in a more ontological direction. Thus, Badiou states: "A world, as a site of being-there, is a Grothendieck topos" (2009, page 295). This would seem to suggest a more ontological use of topos theory. However, Badiou does not elaborate on this point and, although one could read Badiou's discussions of "worlds" at certain junctures of the book along the lines of a topos-theoretical ontology, one can also read them along the lines of topos theory as explaining only "the plurality of possible logics". Indeed, such a reading appears to be more cogent given many specific statements and the overall argument of the book. Thus, even as he distinguishes between *Being and Event*, as placed "under the condition of the Cantor-event and of the mathematical theory [ontology?] of the multiple", and *Logics of Worlds*, as placed "under the condition of the Grothendieck-event", Badiou still conjoins this event with the *Lawvere*-event. This is not incorrect, but Badiou speaks in this conjunction only of "the *logical theory* of sheaves", rather than the *ontology* of sheaves or topoi, more proper to the *Grothendieck*-event (page 38, emphasis added). Badiou also says: "what we are attempting here is a *calculated phenomenology*. ... The usage of mathematical formalisms in *Logics of Worlds* is very different from the one found in *Being and Event*. The difference is the one between being-qua-being, whose real principle is the inconsistency of the pure multiple (or multiple without One), and appearing, or being-there-in-a-world, whose principle is to consist. We can also say that this is the difference between *onto*-logy and onto-*logic*" (pages 38–39, emphasis added). "*Appearing*, or being-there-*in*-a-world", and not "being-*qua*-being", onto-*logic* and not *onto*-logy!

Topos theory is barely discussed in the book and is used mostly implicitly. There is an extensive relevant note, which, however, contains only a bibliography of technical works on topos and category theory (page 538). Both the subsection to which the note is appended and the note itself deal with phenomenological – logical argumentation rather than with ontology, as the titles of both the subsection, "Function of Appearing", and the overall section, "Atomic Logic" indicate (pages 243 – 245, 537 – 539). In sum, topos theory as providing *logics* appears to dominate the argument of *Logics of Worlds*, while *worlds* appear to conform to the underlying ontology that is still set-theoretical, if differently oriented for each world. I shall not, however, attempt to sort out Badiou's ultimate view on the subject. Instead, I would like to explore what an ontological deployment, closer to Grothendieck, of topos theory has to offer vis-à-vis the use of the theory as logic à la Lawvere in Badiou.

As I argue here, in Grothendieck's hands, topos theory is not in the service of explaining "the plurality of possible *logics*". Instead, it *inscribes* the plurality of possible ontologies. Some of these ontologies are forms of the One (possibly the multiple-One), and others are forms of multiple-without-One, including those defined inconsistent multiplicities. There is no single ontology that can encompass this multiplicity of multiplicities in the way the set-theoretical ontology, the multiple of multiples as it is, does for Badiou. Grothendieck's topos-theoretical multiplicities may be topoi of sets, but they need not be. All of these multiplicities are equally actualizable mathematically, at least in principle, and there is no special reason to uniquely prefer any one of them, say, one or another form of set-theoretical ontology in one or another orientation, although conditional preferences are, of course, inevitable. A given ontology, set-theoretical or other, can be given a plurality of logics, while a given logic may be differently inscribed ontologically; and one has a particular combination of logic and ontology, or being and appearance, and hence also of the structure of 'events', in each case. Grothendieck's topos-theoretical ontology is a kind of multi-universe. There may still be a *univocity* of being, which brings being and appearing together, and inscribes events, as Badiou wants, but only within each given ontology, amidst the plurivocity of such ontologies. One must indeed speak, with Badiou, of "logics of worlds", both in plurals. But, as against Badiou, the plurality of "worlds" of multiple ontologies becomes at least as crucial as the plurality of logics.

This richer conceptual architecture is also important pragmatically. As indicated earlier, topos theory arose from the particular concrete mathematical task Grothendieck confronted, the proof of Weil's conjectures in algebraic geometry. It is true that as a logical theory, "*topos* theory defines the conditions beneath which it is acceptable to speak of a universe for thought, based on the *absolutely impoverished* concept of relation-in-general", as Badiou says (2006, page 166, emphasis added). As emphasized throughout this paper, Grothendieck's topos theory, too, defines mathematical objects, beginning with points, by their relations (morphisms or arrows of category theory) to other objects. In Grothendieck's hands, however, this "impoverished" (categorical) concept of relation-in-general serves the ontological task of an *enrichment* of objects of certain categories. Any given object, even a point, becomes multiple, endowed with a multiplicity, because it is defined by arrows, morphisms that link this point to other objects. This enrichment is accomplished by the topos-theoretical geometrization, which is, again, also an *algebraization* of spatiality by virtue of, or indeed defined as, topos-theoretical relationality. This was how Grothendieck brought together and jointly extended algebraic topology and algebraic geometry, first, via the concept of a sheaf (used in both fields previously). A sheaf is a particular kind of arrow space, $Y \Rightarrow X$, over (projected on to) a given space, X, associating a space A, to each point of X, which is why it is called a 'sheaf', a sheaf of spaces over a given space, which can, again, be a single point. By making each topos a whole *category* of sheaves and thus *spaces* (plural) over and indeed defining a given space, the concept topos 'multiplies' this concept into an immensely rich architecture, again, even if X is a single point.

Grothendieck's approach thus entails a general philosophical principle, which may be called the enrichment principle. This principle is defined by the task of giving or associating with structurally impoverished objects of one category the kind of structural richness comparable to that associated with an analogously defined object of another category that is structurally rich in an analogous sense. One might say that it is the task of, as it were, transferring or translating a structural richness from one category to another. As explained above, topos theory enabled Grothendieck and his coworkers to endow, in accordance with Weil's conjectures, certain algebraic varieties, topologically impoverished in the standard topology, with a richer topology and, correlatively, with richer algebraic structures, (sheaf) cohomology groups, associated with them, by constructing suitable functors between the corresponding categories. To do so required an extraordinary technical virtuosity of both abstract and concrete nature.

The situation may, again, be illustrated by modern urban geography. One may, for example, be able to understand better the economic, political, and cultural dynamics of small towns, and even villages, by rethinking this dynamics in terms of the organizational complexity generally associated with large cities, and previously seen as inapplicable, missed in the case of smaller urban formations. That does not mean, however, that such a categorical transfer of operational structures (from one type of urban entities to another) is immediate or easy. For while such richer operational structures are in fact at work in smaller urban formations, they may work quite differently there and are not easy to analyze or to perceive, in the first place, or (when one is also concerned with practical implementations of such new dynamics) to enact.

Grothendieck's enrichment program of algebraic geometry could also be seen as developing and practicing the mathematics defined in one category (in this case, that of topological manifolds) in another category (that of algebraic varieties). The result is a kind of translational experimentation, or experimental translation. One might see Descartes's analytic geometry as the first example of this kind or at least the first step on this road. One can in principle think of practicing any given mathematics in any given ontological domain by thus experimenting with ontologies themselves, since one changes a given ontology by this new practice as well, as again, Grothendieck did in the case of algebraic varieties. That does not, of course, mean that this practice is easy, quite the contrary, as just noted, or that it is only a matter of translation (which is not easy either). It is an invention, more like translating poetry, which is only done well by good poets. New things emerge in the process as well, and a translation of this type is only a starting point; theorems still need to be proven in a new domain, and new concepts invented. It is, however, a kind of "experimental mathematics". The term has been lately in vogue, used more by analogy with natural sciences, such as physics, and there is even a journal called *Experimental Mathematics*. Here, however, the term is given a different sense, in particular that of translational experimentation *with* rather than only *in* mathematics and hence also with mathematical ontologies. The term is used here more in the sense of experiment in art than in science, where experiments are hardly devoid of this artistic sense either. From this viewpoint there is, again, no a priori reason to select any given mathematical ontology in favor of any other, and a new ontology can emerge in a given experiment. Leibniz's God selects one world among many possible ones. In physics one might need a single material ontology (although there are alternative views as well). In the domain of thought things are different, however, and this is where mathematics, or philosophy, lives: mathematics is, I agree with Badiou, a thought rather than merely logic. In accordance with Badiou's approach as well, the (axiomatic-like) decision of thought may be determined by the richness it enables (*BE* 2006, page 96). One need not, however, settle, as Badiou does, for the set-theoretical ontology or center on the set-theoretical view of a given ontology, say, that of topological spaces or algebraic varieties. One can instead develop new ontologies, by relating, by means of functors, different categories.

How does this type of thinking work beyond mathematics? And, even if this thinking does offer ontological advantages over the set-theoretical ontology in mathematics itself, does it offer significant advantages over Badiou's set-theoretically ontological thinking beyond mathematics? I would argue that it does, and I shall, in the remainder of this section and in the next section, outline (I can do no more within my limits here) why such is the case.

For Badiou—in the shadow of Marx, a shadow extending long over Badiou's work—an event still appears to be a product of necessity, just as a political revolution is and its model. Being defies the logic of a previous situation and demands a new one (within, for Badiou, the same set-theoretical ontology), which entails a set of revolutionary decisions. One can, I think, argue for this determination of an event in Badiou, even though an event is always a discontinuous occurrence (vis-à-vis its situation) and its ultimate efficacity is no longer defined only by the set-theoretical ontology but also by the (topos-theoretical) logic of the situation. This type of pressure is important in shaping the eruptive emergence of an event, although the latter comes via a complex multitude of trajectories. Such a pressure, in the form of Weil's conjectures, was a significant factor in Grothendieck's invention of topos theory, a major event in the history of mathematics. And yet topos theory was also a product of experimental translation or transfer of both mathematics and ontologies between categories. One can also think of cases where experimentations led to a new ontology in mathematics, or physics (or other natural sciences) and philosophy, and perhaps especially in art, under a lesser pressure of necessity, although the efficacity of experimentation is rarely, if ever, certain: Choice? Necessity? Both? Something else altogether?

One might call this approach 'ontoexperimental', also by way of juxtaposing the term to 'ontotheological', which has been used by both Heidegger and Derrida, as roughly equivalent to the multiple-One (in juxtaposition to the multiple-without-One) in Badiou. Whatever the balance of experimentation and pressures in Grothendieck's own work may have been, topos theory, as ontological theory, enacts the logic and ontology of *experimentation* in mathematics or, by extension, elsewhere, in science, art, philosophy, or politics. Of course, our ontological constructions, when they are possible, are enabled and constrained differently in different fields, albeit it not entirely differently since these fields may share both enabling and constraining factors. Thus, in mathematics and mathematical sciences, such as physics, our experimentation is subject to very rigorous disciplinary, for example, logical, constraints. Similar and, sometimes, even the same constrains are not absent in other domains, such as philosophy, art, or politics. However, these constraints are often, although not always, more localized outside mathematics, even in physics (which, although mathematical in character, is not quite the same as mathematics) and especially in philosophy, art, and politics. This does not mean that we do not need to be rigorous in these fields; we cannot afford not to be. We need rigorous experimentation, in which rigor, too, can be experimented with, and, against Badiou, rigor cannot always be mathematical, or only mathematical, even in mathematics itself.

4 Ontoexperimental politics and geo-topology of culture

Grothendieck's, roughly anarchist philosophy and politics are not incompatible with the ontoexperimental landscape of his topos theory just outlined, a kind of multi-topos topology.(2) Grothendieck's political thinking and actions, both in general and as concerns the politics of contemporary mathematics and its relation to state politics, is a complex and controversial subject. It certainly cannot be treated in summary fashion, even leaving aside the Pandora's Box of the term 'anarchism'.(3) In fairness, the Marxist dimensions of Badiou's thought would need a more sustained and careful discussion as well, and the term Marxism, too, is a Pandora's Box of its own. Although I would stand by my point concerning the concept of event as primarily the product of pressure and Marxist aspects of the concept, this concept is, as we have seen, manifestly richer and more general. Also, while Badiou defines his vision in *Logics of Worlds* as "materialist dialectic" (as against "democratic materialism", dominant in the current intellectual and political landscape), which, as Badiou acknowledges, carries Marxist colorations, he also qualifies it. Be it as it may, Grothendieck's and Badiou's politics are beyond my scope here. I would like instead to briefly explore a few political implications of Grothendieck's ontoexperimental approach, which this approach has regardless of Grothendieck's own politics.

Badiou's mathematical ontology of the multiple-without-One has important ethical and political implications and is developed with these implications in mind. In particular, the ontology of the multiple-without-One makes ethics irreducibly multiple and hence irreducibly political. The political determination of the ethical as irreducibly multiple is, as it were, the ethical (or, again, political) "categorical imperative" of Badiou's thought, which compels him to move, most expressly in *Ethics* and *Metapolitics*, against Kant's ethics and the tradition that follows extending to Levinas and beyond.(4) Badiou's argument is in part defined by the possibility of mappings between mathematical and political *logics* or *orientations*, governing the set-theoretical ontology of the multiple. Three such orientations in particular are developed in *Being and Event.* They are summarized in *Briefings on Existence* as follows: the first is "constructivist", which "sets forth the norm of existence by means of explicit constructions"; the second is transcendent, which "works as a norm for existence by allowing what we shall coin a 'super-existence' ", ... whereby "every existence is furrowed in a totality that assigns it to a place"; and the third is generic, which "posits existence as having no norm, save for discursive consistency", whereby "all existence is caught in a wandering that works diagonally against the diverse assemblages expected to surprise it" (2006, page 55). According to Badiou:

> "These orientations are—metaphorically—of political nature. Positing that existence must show itself according to a constructive algorithm, that it is predisposed in a Whole, or that it is a diagonal singularity: all of these orient thought according to a repeatedly particular meaning of what it is. And consideration of the 'what is'

(2) Not coincidentally or, as his biographical writings make apparent, inconsequentially, Grothendieck also has an anarchist family background: both his parents were anarchists.

(3) The literature on the subject is extensive and difficult to coordinate in view of the biographical complexities and controversies surrounding Grothendieck's later years after he more or less left institutional mathematics at the age of forty-two in protest against the military funding of the Institute of Advanced Scientific Studies. There are, however, numerous mathematical, biographical, and political writings from this later period, many of which are not yet published. His previous (published) mathematical output is enormous. For a useful background and references, see Allyn Jackson's "*Comme Appelé du Néant*—as if summoned from the void: the life of Alexandre Grothendieck".

(4) I have discussed the subject in detail in "Badiou's Equations—and Inequalities" (2007).

is that of which there is a case, or 'what is' is a place in a Whole; or 'what is' is subtracted from what is a Whole. ... We could transpose what I am speaking in terms of a politics of empirical particularities, a politics of transcendental singularities and a politics of subtracted singularities, respectively. In a nutshell, they are embodied, respectively, as parliamentary democracies, Stalin, and as something groping forward to declare itself, namely, generic politics. The latter suggests a politics of existence subtracted from the State, or of what exists only insofar as it is undecidable.

What is ... wonderful is how these three orientations can be read mathematically by sticking to Set Theory. Gödel's doctrine of constructible sets gives a solid base to the first orientation; the [Cantor] theory of large cardinals provides one for the second orientation; and the [Cohen] theory of generic sets lends itself to the third" (2006, page 56).

The third orientation—defined by the aporethic wandering of political thought, just as the parallel orientation in mathematics is, as discussed in section 1, defined by the aporetic wondering of mathematical thought—appears to be the one that Badiou favors, as concerns both his preferred logic of set-theoretical ontology and his preferred mode of political practice. There is nothing wrong with this preference, or with Badiou's set-theoretical mappings of these three orientations themselves. These mappings are both elegant and effective. The question is, however: How many more orientations or logics could one have under the conditions of the set-theoretical ontology? How many orientations could this ontology allow for? Or, to begin with, is there only one set-theoretical ontology, or do these orientations correspond to different set-theoretical ontologies? Can we really hope that the ontological–logical structures of set theory, no matter how many of those might be, are sufficient to map the political multiples-without-Ones (plural)? The ethical, I agree with Badiou, is a part and a consequence of the political, although a reciprocally shaping consequence, insofar as ethically demarcated areas of the political are possible and sometimes necessary, and may have political effects. What about, for example, Derrida's conception of the politics of "the democracy-to-come", introduced appropriately in *Specters of Marx* (2006)? What about Deleuze and Guattari's politics of *Anti-Oedipus* (1995), or Deleuze's subtle politics of (more) personal resistance in "Postscript on the societies of control" (1997), or politics defined by Foucault's ontology/ies of power? All of these are, I would argue, ontologies of the multiple-without-One. Could they be read 'mathematically' via set theory, however oriented? It is doubtful. Does this make them any less important to understand, for example, in order to decide which of them we want to practice? It is equally doubtful that it does. Indeed, how much of the *political* orientations listed by Badiou can his set-theoretical ontology encompass? Are not these orientations, too, likely to overflow into ontologies other than set-theoretical?

It seems to me—this is, I would argue, a problem of, or for, Badiou's philosophy—that the multiple-without-One as the set-theoretical ontology is not rich enough to do the job here. Nor is any mathematically configurable ontology, that of topos theory included. Beyond mathematics, and in view of Gödel's theorems and related findings even in mathematics, ontology does not appear to be reducible to mathematical ontology, as Deleuze and Guattari suggest in their discussion of Badiou (1994, pages 151–153). In *A Thousand Plateaus* they are compelled to appeal, specifically in the case of the smooth and the striated (most political spaces are combinations of both modes), to different, while interactive, models—mathematical, physical, musical, aesthetics, and so forth—implicitly to a "thousand" models (1987, pages 474–500). In some situations spaces defined by these models form a topos-like architecture, as, say, in the case of Pierre Boulez's music, where the interplay of the smooth and the striated is defined by the topos-like interactive architecture of musical, literary, mathematical, and philosophical spaces rather than taking place in a musical space alone (Deleuze and Guattari, 1987, pages 477–478). Importantly, in some cases, the simultaneous functioning of these models is found and even defines a given 'situation' or 'world', sometimes through its interactions with other situations and worlds, their ontologies and logics. Indeed, this horizontal interactiveness between situations or worlds is sometimes missing in Badiou's analysis, for example, in *Logics of Worlds* (2009), where both ontology and logic of the situations or worlds considered are sometimes too self-contained. Parallels, especially historical parallels, are brought into play (eg, on pages 20–25), but interactions between different worlds on the same stage are rarely considered.

Nor indeed is mathematics itself reducible to mathematics: it always exceeds itself. The multiple-without-One in mathematics is more multiple than mathematics, even in mathematics itself. That, as I said, does not mean that the corresponding ontology or logic is not rigorous. It does mean, however, that other forms of rigor are possible and necessary, even in mathematics. Badiou does not appear to see Gödel's or Cohen's findings in this way, although they do point in this direction, as in fact do already some of Cantor's findings. Also, even though Badiou's concept of event exceeds the set-theoretical ontology, he, as discussed here, still wants to bring this concept and this excess into the mathematical fold by making the logic of an event mathematical logic, defined by topos theory as *logic*. In the present view the mathematics of topos theory, as *ontology*, suggests the irreducible nonmathematical excess even in any mathematical ontology, let alone in ontologies found elsewhere.

One might, accordingly, want to rethink the political and the spaces of the politics, or the politics (plural) of space, on the *model*, rather than strictly in terms of, Grothendieck's ontoexperimental thinking in topos theory, since these spaces can very rarely be (ontologically) mapped mathematically (let alone only set-theoretically), and then only in a limited fashion. As explained throughout this paper, this model defines any spatiality or ontology sociologically and hence when it comes to cultural spaces, politically by relations of a given space or ontology with other spaces or ontologies. At stake is a new, experimental onto-*topology* and thus, also, geo-*topology*, which defines the *architecture* of these relations, rather than only geo-*graphy*, which conventionally maps these spaces, although such mappings, too, have their place in this geo-topology of culture.

As Marx famously said in his final thesis on Feuerbach: "The philosophers have only interpreted the world, in various ways; the point is to change it" (1978, page 145). Marx, however, was more of a thinker of historical necessity rather than of experimentation, as was his younger contemporary Nietzsche, not a thinker especially favored by Badiou, while he is by Deleuze and Guattari, who indeed wanted to bring Nietzsche and Marx together. In contrast to Marx and Badiou (his appeal to "decision" tempers this contrast, while his appeal to dialectic in *Logics of Worlds* sharpens it), one wants to survey a much greater multiplicity. It is the multiple of multiples-without-Ones of actual and possible worlds, of being and appearance, of ontologies and phenomenologies, or logics. We might need to live in many worlds at once, to be many monads at once, to be "plurads" and make our decisions amidst this more radical plurality of many multiples-without-Ones rather than with a more contained plurality, that of one multiple-without-One, of Badiou's set-theoretical ontology. Rather than follow a single or multiple-One dialectical vision, Marxist or other, we must envision and enact changes in the world (in many worlds), make new decisions of thought, and create new events along many lines. Often we need to move in several directions at once, but we especially need to follow those trajectories that allow us to enrich our situation by transferring into it new architectures.

One might append Marx's formulation by saying "the point is to change the world, *in various ways*", sometimes more revolutionary, sometimes more evolutionary, sometimes both in various directions. Indeed, from this perspective, one might also think, perhaps differently from Marx or by reading Marx differently, of an experimental Marxism or Marxist experimentation. While one might be hesitant to speak of a permanent revolution, one might speak of incessant transformation, a becoming with transformative effects, such as those resulting from decisions. This is not easy, and the minefields are many—and (Badiou is right on this point) true events are rare. But experimentation, a rigorous experimentation that *may* result in an event, need not be rare, notwithstanding its minefields or because of them. Just as with Grothendieck's topos theory in mathematics, the logic of rigorous experimentation elsewhere is the logic of being and thought as grace, a grace under and without pressure, and sometimes grace is more difficult without than under pressure.

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