Proceed with Care: Integrating Predictive Analytics with Patient Decision-Making

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1 Introduction

Predictive analytics holds great promise for data-driven decision-making in healthcare operations. However, it is important to carefully account for biases in observational patient data and operational structure to ensure successful implementation of predictive analytics. This chapter describes two case studies that integrate predictive analytics with patient decision-making in hospitals: one targets personalized interventions to patients, and the other seeks to improve patient flow in complex inpatient ward settings.

In the personalized intervention setting, we consider a machine learning model that uses electronic medical record data to predict which patients are at risk for diabetes. While state-of-the-art machine learning models achieve strong predictive performance, they are known to suffer from causal issues or biases. To address this concern, physicians were presented with a human-understandable explanation of the predictive model. Interestingly, physicians were successfully able to leverage the explanation to discover an unexpected but important causal issue in the data. Thus, caution must be exercised when deploying predictive models to aid decision-making on actual patients; explainable AI presents an attractive bridge for integrating domain expertise with high-performing machine learning models.

In the patient flow setting, we consider a discharge decision-making problem to balance the tradeoff between patient readmission risk and inpatient ward congestion. Even with a predictive tool that is calibrated from observational data and addresses the possible biases, it is still non-trivial to translate the prediction into day-to-day operational decisions, particularly when the environment is dynamic and uncertain. We describe in this case study how to build a system model for the patient flow and how to integrate a personalized readmission prediction tool to dynamically prescribe both how many and which patients to discharge on each day.

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Through these two case studies, we illustrate that predictive analytics alone may not lead to better decisions. It must be implemented in tandem with careful consideration of domain expertise and operational structure.

2 Personalized Interventions

Predictive analytics holds great promise for improving medical decision-making. However, a key challenge is that predictive models are highly specialized to perform well on the data on which they are trained. Yet, for a number of reasons, the training data may not be representative of the data observed by the deployed model. One common reason is that the patient mix may differ significantly across domains. For instance, patients that visit the ICU tend to be far sicker than the general population, so a model that is trained to achieve good performance on ICU patients may perform poorly on general patients. Similarly, different hospitals often have systematic differences in how they code diagnoses for patients; these differences can cause a predictive model that is trained to perform well for patients at one hospital to perform very poorly at a different hospital (see, e.g., [1]).

A more subtle challenge is that predictive models are often trained on *observational data*—i.e., data that is obtained from monitoring existing patients rather than by running a randomized clinical trial. However, these patients are already subject to medical care, which systematically biases the observed data. To illustrate, in one case study, [2] built a machine learning model predicting mortality for pneumonia patients. Oddly, the model predicted that patients with a history of asthma have lower mortality risk than the general population; this is unexpected since asthmatics generally have higher asthmatic risk (if untreated). Yet, the model was not wrong — this pattern was reflected in the observational data due to systematic differences in treatment decisions. In particular, pneumonia patients with a history of asthma were usually admitted directly to the ICU, where they received aggressive care that was so effective that it lowered their mortality risk relative to even the general population. Unfortunately, as a consequence, machine learning models trained on the data incorrectly learn that asthma lowers mortality risk. In other words, even though the model performs well on predicting patient outcomes in the observational data, it is not useful for decision-making since it does not correctly distinguish which patients are in need of treatment.

These challenges are particularly problematic for *blackbox models*, which are models such as deep neural networks (DNNs) and random forests that are difficult or impossible for humans to understand due to their complex, opaque structure together with their use of a large number of explanatory variables. Simple models such as decision trees are much easier to understand, yet they are often outperformed by blackbox models. That is, there has traditionally been a tension between predictive performance (maximized by using blackbox models) and human-understandability to diagnose potential issues or biases in the data (achieved by using interpretable models).

2.1 Explaining Blackbox Models

A promising middle ground is to train a blackbox model, but then leverage techniques to interpret the prediction made by the blackbox model in a human-understandable way. Broadly speaking, there are two kinds of techniques for interpreting blackbox models. The first kind produce *local explanations*, which describe how the model made a prediction for a single input. For instance, suppose we train a random forest model that predicts whether a patient has diabetes based on their demographics, vitals, past diagnoses, and current lab results. For a given patient—say, Bob—a local explanation may say the blackbox model predicts that Bob has diabetes due to his high glucose level. For a different patient—say, Alice—a local explanation may say that the blackbox model predicts that Alice has diabetes since she is currently taking insulin. In other words, local explanations can help physicians understand the reasoning behind a single prediction made by the blackbox model.

In contrast, *global explanations* attempt to explain the high-level reasoning performed by the blackbox model across all patients in a given population. For instance, given the machine learning model trained to predict diabetes together with a dataset of patients, such a technique might approximate the blackbox model using a simpler, interpretable model (e.g., a decision tree). Since the simpler model approximates the blackbox model, we expect that major issues with the blackbox model will also be reflected in the simple model. Thus, a human decision-maker can use the global explanation to detect issues in the blackbox model *before* it is deployed to be used in a real-world setting.

2.2 Case Study

The remainder of this section is based on a case study from [3], which demonstrates how global explanations can be used to diagnose issues in blackbox models. In this setting, the authors sought to build a machine learning model predicting diabetes risk for patients. Diabetes is a leading cause of cardiovascular disease, limb amputation, and other health problems; by preemptively predicting which patients are likely to be diagnosed with diabetes, physicians can propose health interventions such as exercise and improved diet to reduce patient risk. Indeed, clinical trials have demonstrated the effectiveness of preventative interventions in reducing risk for diabetes.

The blackbox model was trained using de-identified electronic medical record data from multiple providers. Each patient was associated with several hundred features extracted from their electronic medical records from previous visits to the healthcare provider in the past three years. These features spanned demographics, ICD-9 diagnosis codes, prescription medications, and lab test results. The binary outcome variable was whether the patient received a type II diabetes diagnosis in their most recent visit to the healthcare provider. This dataset was preprocessed by domain experts to ensure that only information prior to the diabetes diagnosis was available to the predictive model.

The authors first considered data from just the largest provider, which included 578 unique patients. Following standard practice, 70% of the data was used as a training set while 30% was used as a test set. Multiple machine learning models were tested, and the random forest model was found to be the best in terms of predictive performance. However, as noted earlier, electronic medical record data constitutes observational data which may suffer from various biases; thus, a high-performing predictive model may form incorrect conclusions. [3] propose the DTExtract algorithm, which produces a global explanation of the random forest. DTExtract uses active learning to efficiently sample new data points, labels them with the blackbox model, and uses the resulting augmented training data set to construct an interpretable decision tree. In other words, this algorithm constructs a decision tree that *approximates* the random forest. The resulting global explanation of the random forest model is shown in Figure 1.

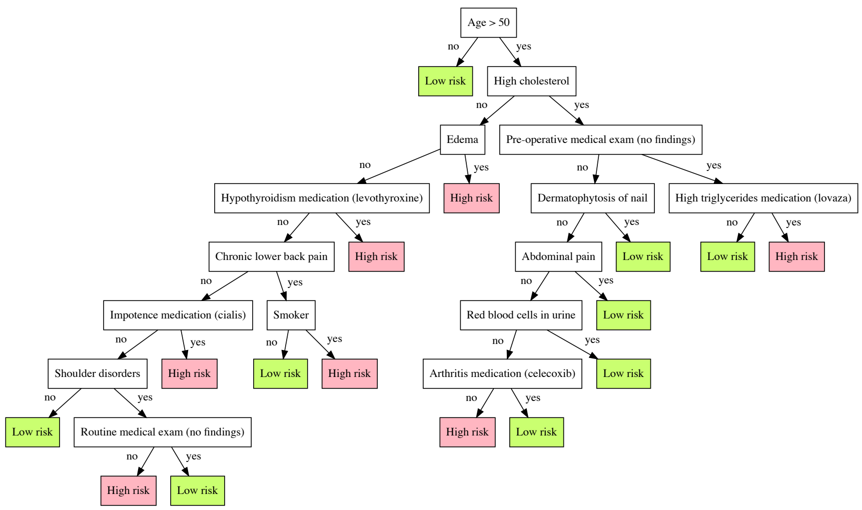


Figure 1: Global explanation for the blackbox random forest trained to predict diabetes for patients in the largest provider [3].

This global explanation was then shown to physicians to see if it was reasonable. They derived a number of insights based on this explanation, some of which suggested possible issues in the underlying random forest.

Bias in Diagnosis for Sicker Patients. Most notably, the model appears to reason about prior diagnoses that are unrelated to diabetes. In particular, consider the subtree of Figure 1 rooted at the node labeled "Dermatophytosis of nail." This subtree applies to the subpopulation of patients who are over 50 years of age, have high cholesterol, and furthermore have not had a pre-operative medical exam. According to the explanation, these factors are all indicators for higher diabetes risk.

However, for this subpopulation, the decision tree predicts that patients with medical diagnoses such as dermatophytosis of nail, abdominal pain, and red blood cells in urine, are *less* likely to have diabetes. In other words, it says that patients who already have other medical conditions have lower diabetes risk. Physicians found this effect to be counterintuitive, since dermatophytosis of nail has no known negative relation to diabetes; if anything, patients with this condition should be *more* likely to have diabetes.

Upon further reflection, the physicians suggested a possible explanation: it might be the case that patients who have these other health conditions are more likely to have visited the physician recently. Thus, they are likely to have received preventative measures to reduce their risk of diabetes. In contrast, patients without prior health conditions may not have visited the physician, and therefore may not have been recommended to undertake preventative measures. Statistical checks on the original random forest model, suggested that it also suffers from the same biases.

As described above, using this random forest to make decisions could lead physicians to underestimate the diabetes risk of patients in this subpopulation, and fail to recommend preventative measures to high-risk patients. The global explanation enabled physicians to diagnose an important issue with using the random forest. Once discovered, such issues can be addressed using standard techniques (e.g., adding the number of recent visits as a control variable).

Comparison to Different Provider. The authors then repeated the same process on data from the second-largest provider, which included 402 unique patients. They trained a new random forest model and used DTExtract to explain it; the resulting decision tree is shown in Figure 2.

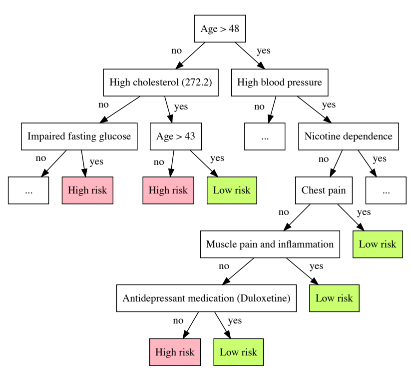


Figure 2: Global explanation for the blackbox random forest trained to predict diabetes for patients in the second-largest provider [3].

This explanation was also shown to physicians, and they were asked to interpret how it differed from the explanation trained on data from the first provider.

A key observation they had is that, unlike the previous explanation, this decision tree includes a diagnosis “Impaired fasting glucose,” which corresponds to the standard lab test intended to evaluate diabetes risk. It appears that physicians at this provider were more active about having patients undergo rigorous glucose tests to identify early warning signs for diabetes. As a result, the two predictive models treated this feature very differently. The predictive model trained for the previous provider essentially ignored the feature (since it was rarely diagnosed, and therefore rarely informative); in contrast, the predictive model trained for this provider significantly benefits from taking it into account.

In general, it can be very difficult for hospital management to discover these types of systematic differences in physician diagnosing behavior or data recording across healthcare providers. The use of such explanations can aid physicians to discover and shed light on these biases.

Non-Monotone Dependence on Age. The authors of [3] note an additional observation made by the physicians about the explanation in Figure 2. The physicians noticed that the dependence on the patient’s age is actually non-monotone. Normally, one would expect older patients to have higher diabetes risk. Indeed, the explanation tends to assign higher risk to patients older than 48 years of age. However, it additionally singles out patients between 43 and 48 years of age with high cholesterol as having high diabetes risk (as opposed to all patients older than 43 years of age with high cholesterol). The physicians hypothesized the following explanation for this effect: high cholesterol is more common in older patients, but for younger patients, its particularly abnormal and therefore suggests a greater risk for diabetes. In this manner, explanations can yield novel data-driven insights that can be tested in the future to further improve patient targeting or care management.

3 Patient Flow Management

The previous case study illustrates the importance of leveraging domain knowledge to correct possible biases in building predictive models from observational data. Yet, even with a properly calibrated predictive tool, it could still be challenging for hospital managers to directly use prediction in solving complicated decision problems, particularly in a dynamic, uncertain environment. Properly accounting for the dynamics in individual patient disease progression and more importantly, the system-level operational perspective is critical when integrating predictive tools in decision-making. In the second case study, we describe an illustrative example integrating predictive and prescriptive analytics to provide a powerful yet easy-to-implement decision support tool in solving a critical issue faced by many hospitals around the world: inpatient discharge management.

3.1 Background

> *A hospitalist makes many decisions that influence the cost of an inpatient stay...but probably none has more impact than "Should this patient go home today or tomorrow?"*
> –Cover story for American College of Physicians (ACP) Hospitalist, October 2014 [4].

Inpatient discharge decision plays an important role in patient outcomes, hospital financial performance, and patient flow management, impacting all care providers from small community hospitals to major teaching hospitals. This cover story further highlights the key tradeoff in making discharge decisions: "Under the Affordable Care Act, it is still in hospitals' financial interest to discharge patients as soon as possible, but also to facilitate post-discharge care and prevent 30-day readmissions. Rather than just lowering LOS (length-of-stay), hospitals now aim to optimize it at the intersection of quality and cost."

In other words, the key tradeoff in making discharge decisions lies at the intersection of quality of care and cost. To alleviate inpatient ward overcrowding, hospitals may discharge patients early; this practice shifts the burden to the early discharged patients, who may experience increased risk of readmission, mortality, and other adverse outcomes. On the other hand, when occupancy levels are low, hospitals may keep patients longer, which can have a positive impact on patient outcomes. How to balance this tradeoff in a dynamic, uncertain environment has broad implications for patient flow, inpatient unit congestion, quality of care, and post-discharge risk.

Currently, most hospitals engage in adaptive discharge practices in a reactive and ad-hoc manner. For example, as illustrated by [5], when a hospital becomes overcrowded, a communication is sent to all physicians asking them to discharge as many patients as possible to free up beds. This unstructured approach may end up discharging too many or too few patients, or discharging a suboptimal set of patients. The authors of [5] note that the individual physicians lacking a system perspective could be one reason why they react to occupancy crises poorly. There is a growing need for analytically guided tools to help hospital managers balance the delicate tradeoff between individual patient outcomes and the system-level ward crowding [6].

3.2 Case Study

The remainder of this section is based on a case study from [7], which develops a decision support tool in discharge management to improve hospital patient flow and reduce readmissions. Along with a data analytics company, the authors have done a pilot implementation of this decision tool at a local community hospital in the State of Indiana.

The first component that the community hospital asked for is a predictive tool of patient readmission risk evolution as a function of her length-of-stay (LOS) in the hospital. A substantial amount of efforts was spent to develop this predictive tool. Similar to the first case study, the authors of [7] also faced challenges from building the predictive tool based on observational data. Specifically, most existing readmission prediction tools treat LOS as an exogenous variable. Directly applying these tools by varying LOS suffers from endogeneity (sicker patients tend to stay longer and have a higher readmission risk), which leads to the incorrect conclusion that extending length of stay for an individual patient results in higher readmission risk. In addition, when applying the classical Cox proportional hazard model to predict readmission timing, there were additional challenges including the excess zero count issue [8] and patient heterogeneity in the readmission timing. [7] integrated several statistical methods and proposed one prediction model that works reasonably well; see Figure 3 for an example of the output from their prediction tool. Each of the curve is the readmission risk "trajectory" of a patient produced from the predictive tool, showing how the readmission probability would change with each additional day that the patient spends in the hospital.

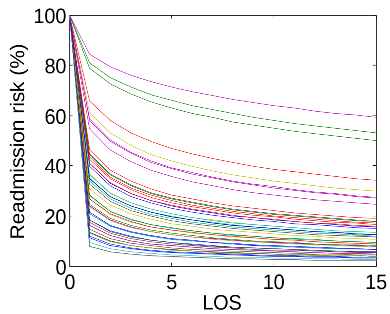


Figure 3: Predicted readmission risk trajectory against length-of-stay (LOS) from [7]. The plot shows the 90-day cumulative probability as a function of LOS (50 random patients selected from the testing data).

Nuances with using the predictive tool in discharge decision. After this predictive tool was developed, hospital management still faced complex decisions on how to turn the prediction into decision-making, in particular, how to use this predictive tool in the day-to-day dynamic environment to decide on how many patients and who to discharge on a given day. Discharge decisions must not only account for the risk of each patient, but also for each patient's risk evolution over future days in conjunction with current and future occupancy levels. The inpatient arrival day-of-week phenomenon further complicates discharge decisions. To explain, consider the following simple illustrative example. Patient A has a relatively high risk currently, but this risk is unlikely to improve significantly by keeping the patient longer. Then the best decision readmissions may be to discharge patient A now. The reverse may be true for a patient with lower discharge risk that may improve significantly by staying one day longer. This contradicts the common industry that a simple risk threshold is sufficient (i.e. discharge all patients when their risk drops below a certain level). In addition, the decisions are modulated by considering current and future occupancy levels rather than risk alone. These complexities necessitate a forward-looking, dynamic approach that cannot be easily intuited.

Accounting for operational aspects. As discussed, the discharge decisions need to account for the current risk of each individual patient in the hospital unit as well as each patient's risk evolution over future days in conjunction with expected future patient arrivals, current and future occupancy levels. To incorporate these important operational aspects – occupancy levels and future arrivals, one needs to build a system model to capture the patient flow dynamics, which then provides the basis for making dynamic discharge decisions.

Figure 4 illustrates a schematic representation for the system model built in [7]. Consider a hospital ward that has a fixed number of beds (denote this number of beds as N, e.g., $N = 50$ beds for a 50-bed inpatient unit). The first box presents the inpatient ward, where the upper arrow coming to the box represents new patient arrivals to this ward – often referred to as a service station in the literature. The patient's hospital length-of-stay corresponds to the time of the patient receiving "service" in this station. The arrow coming from this first box corresponds to the main decision we are considering here: whether to discharge a certain patient that is currently in the box to home. If the patient is discharged, she enters the recovery process represented by the second box in Figure 4. After this recovery process, the patient could be either fully cured with probability $1 - r(LOS)$ and never comes back to the system, or he/she is not cured and needs to be re-hospitalized with probability $r(LOS)$. This readmission probability $r(LOS)$ is from the predictive model and depends on the individual patient characteristics and how long the patient spent in the previous hospital stay (LOS). The readmitted patients come back to the first box, forming a second stream of input to the hospital station in addition to the new patient arrivals.

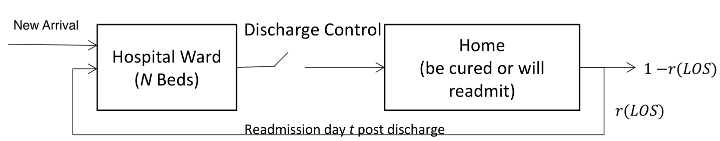


Figure 4: Patient flow model of the hospital ward.

Two key features need to be accounted for when making dynamic decisions based on this system model: (i) each patient may have her own readmission risk trajectory (patient heterogeneity) – it is important and increasingly common to have personalized risk predictions; (ii) the resource – inpatient beds – is limited, which means the decisions to discharge each patient currently "in service" are not independent but connected through the resource constraint. This also corresponds to interplay between the individual patient perspective and the system perspective presented in the discharge problem. To capture these two features and reflect the key tradeoff, one needs to setup an appropriate dynamic decision framework – the output from which is the optimized recommendation on "who and how many" patients to be discharged. A powerful and commonly used tool for such dynamic decision is the Markov Decision Process (MDP) framework.

The first step to setup an MDP framework is to specify the "state variable," i.e., what information that hospital managers would take into consideration when making the discharge decision. For example, how many patients are currently in the system, how long each of these patients already spent, their current predicted readmission risk and future risk trajectory. The next step is to specify the "action" to be taken and the "cost" associated with the action. Action corresponds to the decision to be made at each day, which is the who and how many discharge decision in this current setting. The cost associated with each action is to capture the key tradeoff in a decision problem. Given that the tradeoff we consider in this case study is the inpatient congestion versus patient readmission risk, two cost terms are incorporated: the first is a cost that depends on the number of patients exceeding the bed capacity N, i.e., the overage cost that reflects capacity shortage and ward congestion; the second is a cost that depends on the predicted readmission penalty cost if we discharge a certain patient. Clearly, discharging many patients too early reduce the first congestion cost but increase the second readmission cost, while discharging too few patients increase the congestion cost but reduce the readmission cost. This tension is exactly why one needs to find an *optimal* decision each day.

In a typical MDP framework, the decision-maker is often assumed to aim at minimizing the average cost over a certain period of time (e.g., a week or a month), not just the current day. For example, hospital managers may want to proactively discharge patients in anticipation of a large volume of arrivals showing up the next day. Myopic decisions that only focus on what happened today is often suboptimal. To account for future periods, one need to describe how the state would transition after an action is taken today. In this discharge setting, the transition dynamic is in fact simple: those who are not discharged today will stay in the hospital tomorrow, with their LOS being updated by one day (readmission risk is updated accordingly); new patients arriving today will occupy a bed or wait for a bed depending on the capacity and current occupancy level. Readers are referred to [7] for the mathematical representations of the state, action, cost, and the state transition dynamics.

Who and how many to discharge. Once the MDP framework is formulated, solving the MDP is often non-trivial with standard methods such as value or policy iteration. A main reason is that there are often millions of states or actions to consider for a realistic size problem. To overcome this well-known curse-of-dimensionality, one often needs to resort identify nice properties in the optimal action structure and leveraging approximation methods.

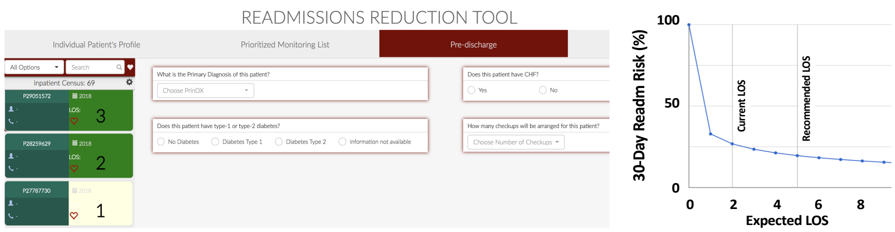
For the discharge problem, an interesting finding from [7] is that there is a priority ranking in terms of the readiness to discharge. That is, the optimal policy will discharge all patients of a higher readiness before discharging any patients of a lower readiness. The ranking of the readiness depends on *marginal* change in the readmission risk between today and a future day, not the absolute value of the readmission risk. At a high level, when deciding between two patients on who to discharge, since the congestion cost they cause is the same (as each occupying one bed), it is preferable to discharge the patient with a smaller marginal change in the readmission risk between today and tomorrow. This priority ranking also answers the "who to discharge" question.

Regarding the "how many to discharge" question, the intuition is to discharge more patients when the occupancy is high and less when low. However, computing the optimal number is much more complicated. The authors of [7] leveraged an approximation for the cost-to-go for all future periods based on the exact solution of the main problem for a quadratic cost structure. The closed-form solution from this quadratic-cost problem preserves the aforementioned structural properties on the priority ranking and leads to a univariate optimization. The univariate optimization is not only computationally efficient for implementation, but also allows one to incorporate full patient heterogeneity into decision-making and is robust to adapt to complex hospital environment such as time-varying arrivals on different days of a week.

Tool implementation and Results. To demonstrate the value of such an analytically guided tool, [7] developed a counterfactual study using a trace-based simulation to compare how the hospital would have performed using their discharge tool versus what the hospital actually did in the historical data. They showed that the dynamic policy produced from their MDP framework could significantly reduce readmission risk for medium- and high-risk groups (from 32% to 28%) when extending the LOS slightly (from 3.33 to 3.55 days). The dynamic policy was also able to correctly recommended intervention (i.e. extending LOS) on over 50% of the patients that were readmitted in the data set.

Further, using an extensive simulation analysis, they showed that the dynamic policy produced from the MDP framework always dominated a *static policy* that also used the predictive information. The improvement gained by the dynamic policy is particularly significant for smaller hospitals and patients with shorter average LOS. This finding again shows that, even with a predictive tool, it is nontrivial to use the tool properly, particularly in a dynamic, changing environment that has many uncertainties. A sophisticated dynamic decision support tool is often necessary.

On the other hand, the MDP framework, though sophisticated, was designed for easy integration with hospital IT system and EHR. Together with their partner hospital and a data analytics company, [7] tested and implemented a cloud-based tool to provide discharge decision support. Figure 5 shows a screen shot of the tool, which has recently been integrated into the hospital's IT infrastructure and provider workflow. The tool displays (a) patients currently in the hospital unit (represented by each block), ranked with different color codes in terms of their discharge readiness (the priority ranking mentioned above); (b) discharge risk curve for future possible LOS of each patient (with past LOS and generally recommended LOS).



(a) Screenshot of main portal

(b) Individual risk prediction tab

Figure 5: Screenshot of the discharge decision support web portal implemented in the partner hospital of [7].

4 Conclusion and Additional Literature

Through the two case studies, we illustrate that caution must be exercised when integrating predictive analytics with patient decision-making. In the first case study, we demonstrate the importance of model interpretability and domain knowledge in building predictive tools from observational data. Tools from interpretable machine learning are critical to ensure that we do not entrust faulty or biased predictive models with patient decision-making. In the second case study, we further show that it is nontrivial to apply a readily-developed predictive tool in complex decision-making. The prediction-decision framework in the second case study has the potential to overcome many common challenges faced more broadly in risk prediction in healthcare and other fields. Combining both of these approaches paves a new road for personalized dynamic decision problems, which are becoming increasingly necessary in healthcare and other services industries. Potential applications include chronic disease management, adverse event prediction for hospital inpatients due to conditions such as deep vein thrombosis screening and sepsis, and proactive interventions for adverse events in the elderly population.

Additional Literature. Many have studied directly building interpretable predictive models via rule lists [9] or decision trees [10, 11]. Yet, blackbox predictive models often continue to outperform interpretable models on a range of predictive tasks. Consequently, a literature has emerged interpreting blackbox model predictions. While the first case study discussed DTExtract for approximating a blackbox model, other methods seek to better understand different aspects of the model. For example, LIME learns an interpretable model locally around a given prediction [12], allowing one to reason about a particular patient's prediction; saliency maps perform a similar role for deep neural networks [13]. SHAP assigns each feature an importance value for a particular prediction [14]. However, recent literature warns that the resulting interpretations may be misleading since they identify correlations rather than causal effects [15]. Alternatively, if one is concerned about data corruption or outliers, [16] uses influence functions to identify (potentially problematic) training points that are most responsible for a given prediction.

Discharge management to improvement patient flow has received much attention in the operations management literature. [17] is one of the earliest papers to study the tradeoff between discharge risk and inpatient occupancy. The authors focus on steady-state performance analysis under two fixed policies (with and without early discharge). [18] consider the scenario when a new patient arrives to a full ICU, doctors must decide which patient to discharge to free a bed. [19] consider the joint decision of ICU admission and discharge decisions, where the decision-maker determines whether to admit an arriving patient to the ICU or to the general ward and also who to to discharge early if a patient needs to be admitted to a full ICU. [20] study the joint problem of coordinating elective case mix and discharge policies. They find that coordination has benefits over a siloed approach when costs of either the operating theatre and/or inpatient beds are sufficiently high.

For tackling high-dimensional MDP problems with large state or action size, Approximate Dynamic Programming (ADP) is a powerful technique ; see [21, 22] for details and the references there. The approximation used in [7] is connected to the broad literature of value function approximation, i.e., approximating the value function by a linear combination of basis functions. Commonly used methods for value function approximations include temporal-difference learning method [23], Q-learning [24], and linear programming approach [25, 26]. Policy-gradient based methods [27, 28] help address issues with large action space.

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