A Threshold Model of Social Contagion Process for Evacuation Decision Making

Samiul Hasan[1]

Purdue University, West Lafayette, IN 47907, USA

Satish Ukkusuri[2,*]

Purdue University, West Lafayette, IN 47907, USA

Abstract

Individual evacuation decision is often characterized by the influence of one's social network. In this paper a threshold model of social contagion, originally proposed in network science literature, is presented to characterize this social influence in evacuation decision making process. Initiated by a single agent, the condition of a cascade when a portion of the population decides to evacuate has been derived from the model. Simulation models are also developed to investigate the effects of community mixing pattern and initial seed on cascade propagation and the effect of previous time-steps considered by the agents on average cascade size. Insights related to social influence include the significant role of mixing pattern among communities in the network and the role of initial seed on cascade propagation. Specifically, faster propagation of warning is observed in community networks with greater inter-community connections.

1. Introduction

Over the last decade, the study of large-scale network systems spanning from the Internet to social networks has grown enormously (Newman, 2003a;

*Corresponding author

Email addresses: samiul.hasan@gmail.com (Samiul Hasan), sukkusur@ecn.purdue.edu (Satish Ukkusuri)

[1]Ph.D. Student, School of Civil Engineering

[2]Associate Professor, School of Civil Engineering

Boccaletti et al., 2006). Understanding the coupled dynamics between the structural properties and the functions of networks has been the principal focus of a wide area of research. Due to the ubiquitous presence of networks, the understanding of the complex network dynamics can have applications in many fields such as the Internet and communication systems, infrastructure systems, logistics and supply chains, trade markets and financial systems, biological systems, and social organizations.

"Network science", an emerging research field, brings an interdisciplinary view to the study of *complex networks* (i.e. networks having an irregular, complex and dynamic structure). This new research field shifts the focus of the studies of networks from the analysis of single small graphs to the consideration of statistical properties of large-scale real network systems (Newman, 2003a; Albert and Barabasi, 2002). Studies on these large-scale real networks have produced many new concepts and measures attempting to characterize the structure of networks. A series of unifying principles and statistical distributions related to different properties of real networks are identified from theses studies. One of the foremost discoveries in this regard is the existence of *small-world* property in many real networks (Milgram, 1967; Travers and Milgram, 1969; Watts and Strogatz, 1998). The small-world property refers to the fact that despite their large size most networks have relatively short path between any of their two nodes. Another relevant property relates with the degree of a node (k), i.e, the number of its direct connections to other nodes. In real networks, the degree distribution $P(k)$, defined as the probability that a randomly chosen node has degree k, is significantly different from the Poisson distribution that has been traditionally assumed for modeling a random graph. Instead studies have found that, in many cases, real networks exhibit a power law (or scale-free) degree distribution (Barabasi and Albert, 1999). Moreover, most of the real networks are characterized with higher density of triangles (e.g., cliques in social network where every member knows every other member) than a comparable random network. Real networks also exhibit significant correlations in terms of node degrees or attributes.

These discoveries initiated a substantial amount of research efforts on developing new network modeling tools for understanding the structural network properties and subsequently reproducing the structural properties observed from empirical network data and for designing such networks efficiently. It is naturally expected that the structure of a real network will have an impact on its functional behavior. Therefore network models were moti-

vated to a better knowledge of the evolutionary mechanisms responsible for the growth of the network, and thus a better understanding of its dynamical and functional behavior. It is indeed found that real networks have interesting coupling between its architecture and functional behavior. This coupling has consequences on the robustness of a real network and its responses to external perturbations characterized by random failures or targeted attacks (Albert et al., 2000).

This coupling architecture of complex networks also has the potential of studying the emerging dynamical behavior of a large number of entities interacting through complex topologies. For example, a fundamental question relevant to complex network systems is how the interactions between the nodes or vertices may cause new ideas, information or fads to propagate throughout the entire network. When does the propagation of such information become a global cascade? How quickly does it spread? To what extent? Such phenomenon are called as *cascades* or *contagion* and are common phenomenon in many real world systems. For example, in the transmission of infectious diseases through communities in biological systems (Murray, 2002; Anderson and May, 1991), global spread of computer viruses on the Internet network (Newman et al., 2002; Balthrop et al., 2004), diffusion of activities, beliefs, ideas, and emotions in social networks (Coleman et al., 1966), power grid failures in electricity markets (Kinney et al., 2005; Sachtjen et al., 2000) and the collapse of financial systems (Sornette, 2003).

The characterization of contagious behavior is pertinent and plays a vital component to investigate transportation demand analysis under specific situations. Since transportation systems, have a significant coupling between the dynamic demand manifested by the complexity of human behavior and the dynamic supply, manifested by the significant variations in network characteristics, small changes in the behavior can significantly impact the transportation network. The goal of this research is to develop a novel model for understanding the cascade of the warning information flow in the social network during hurricane evacuations. The model is developed using emerging notions from complex network science. In addition to model development, several numerical simulations are conducted to demonstrate the spread of warning information during hurricane evacuation.

Hurricane evacuation is often characterized as a complex process involving decision making at different levels-individual, household level influences and community level influences. Whether to evacuate or not; when to evacuate, where and which route to take are some important dimensions of the

decisions involved in the evacuation process. Individual's evacuation behavior primarily depends on three basic social psychological processes: risk perception, social influence and access to resources (Riad et al., 1999). Studies on hurricane evacuation (Baker, 1991, 1995; Dash and Gladwin, 2007) found that in addition to the factors such as individual and household characteristics, risk level, evacuation orders, and storm threat, the personal risk perception was the most important factor in determining the evacuation decision. Although the role of social influence on risk perception behavior is not directly addressed in the literature, one can suspect that the individual risk perception can be socially influenced as evacuation decisions spread through social networks.

During evacuation, in addition to personal risk perception, social influences play an important role on individual's decision making process though individuals are finally responsible for their own decisions (Riad et al., 1999). Evidence of the influences of individual's social network on evacuation decisions were found in studies (Riad et al., 1999; Clifford, 1956; Perry, 1979; Mileti and Beck, 1975; Drabek and Boggs, 1968; Quarantelli, 1985). Social relationship of an individual can be thought of a combination of kin relationships (i.e. relatives) and community contacts (i.e. friends and neighbors). Both kind of relationships have influences on warning propagation and evacuation decisions. Previous research suggests that individual's social ties have an impact on the disaster warning dissemination (i.e. content, source, and number of warnings received) and adaptation process (Clifford, 1956). It is found that the greater the number of contacts and ties one has to the community, the more likely one is to receive information on evacuation recommendation (Perry, 1979). Official warning messages sometimes provide vague information that are usually confirmed through other sources (i.e. through individual's social network) (Mileti and Beck, 1975). It is however generally agreed that kin relationships play more important role compared to community relationships in evacuation decision making. Perry (1979) however reasoned that when kin relationships are weak or absent, community contacts can serve similar function with respect to a model of evacuation behavior.

Characteristics of individuals' social network can be predictors of evacuation patterns. For example, it is found that individuals who do not typically evacuate have a small social network and vice versa (Drabek and Boggs, 1968). Previous social science studies on hurricane evacuation also suggest that African American households typically possess more cohesive kinship

and larger community networks compared to Caucasian communities and hence have greater propagation of disaster warning information (Perry, 1979). Previous research also find evidence that ethnic groups such as African American households actively involve their elders within the kin network; which eventually contributes to a higher percentage of decisions to evacuate (Quarantelli, 1985).

However, in spite of the qualitative insights from many social science studies about the impact of social network on hurricane evacuation, a model which can characterize the complexity of the influences is currently lacking. Within this context, in this paper, we propose to develop a novel network science model to specifically investigate the social influence process within a community network. In order to model this social influence process we adopt the *threshold model* of social contagion process proposed by Watts (2002). Watt's original model is motivated by a context where individuals in a population must decide between two alternative actions, and whose choice of actions depend explicitly on the actions of other members within the population. In other words individuals in a population adopt alternate behaviors by following their peers (e.g. relatives, friends, and neighbors). Such kind of decision making may occur in social and economic systems when decision makers have to pay attention to others either because of limited available information about the problem or because of individual's limited ability to process the available information. In such cases one seeks additional information of the problem or seeks advice to friends, relatives or colleagues or simply makes the decision which most people make.

Although the processes of risk perception and access to resources are well studied within the evacuation behavior literature, the process of social influence is not well addressed. One of the major deterrent factors to empirically observe the social influence process is the requirement of having information on individual's social network. Such kind of information are not readily available in evacuation surveys which are the basis of most of the evacuation studies. Therefore an analytical model is essential to translate qualitative insights into quantitative measures of the social influence process on evacuation behavior. As such the model developed in this paper makes a significant methodological contribution and provides several important insights on the role of social relationships on evacuation behavior. Specifically it intends to determine the relationship between community or social network characteristics and the aggregate evacuation behavior.

2. Model Description

The threshold model of social contagion process mimics a binary decision making context where an agent (i.e. an individual or a household) has to decide between two choices of whether to evacuate or not due to a hurricane threat. The agent follows a simple binary decision rule observing the current states (either 0 or 1 i.e either evacuated or not-evacuated respectively) of k other agents which we call its neighbors, and adopts state 1 if at least a threshold fraction of its k neighbors are in state 1, else it adopts state 0. The neighbors represent the members of the agent's social network (the network of friends, relatives, colleagues etc.). To account for variations in risk perception and access to resources required to evacuate, the threshold value of an agent is treated as heterogeneous. That is the threshold value of an agent is randomly drawn from an arbitrary distribution of threshold values.

Each agent belongs to a particular community having a specific degree (i.e. number of neighbors) distribution. That is the number of neighbors of an agent is drawn from the degree distribution of the corresponding community of the agent. We assume multiple communities in the population and different level of connections within the community and between communities. For example, if individuals prefer to form friendship based on race then the network can be separated into different communities by race. Thus the connection within the community represents the number of neighbors of an agent of same race and the connection between communities represent the number of neighbors of an agent with different races. This mixing pattern in a population can be characterized by a quantity e_{ij} which is defined as the fraction of neighbors in a network that connect an agent of type i to the agent of type j. We assume an undirected social network i.e. $e_{ij} = e_{ji}$.

In order to investigate the effects of the strength of relationship on social contagion process we introduce the weight $w_{ij}$of the edge. In this case an agent does not only consider the fraction of its neighbors, instead each neighbor's weight is used calculate the fraction. A weighted fraction is calculated by dividing the weights of the affected neighbors by total weights of the edges of the agent.

In hurricane evacuation one important event to observe is that when does a significant fraction of the population decide to evacuate. We refer to this event as a *global cascade* where the term *cascade* refers to an event when any fraction of the population decides to evacuate. Thus the term

global cascade refers to a cascade of significantly large size (in practice this represents more than a fixed fraction of the size of the network). In this work, we are interested to investigate the following research questions within the threshold modeling framework of social contagion over a multiple community network :

a) What is the condition for a cascade to occur, in other words under what condition a fraction of the agents decides to evacuate.

b) Social networks with multiple communities can have different kinds of mixing patten in terms of social connections; what is the role of this mixing pattern on the propagation of evacuation decisions.

c) Social contagion process is initiated by switching the state of a single agent from 0 to 1. This single agent (i.e. the *initial seed*) therefore plays an important role over the contagion process. We investigate the role of the characteristics of the initial seed on the propagation of evacuation decisions.

d) In addition to the state of the neighbors, the time when they make their decisions may also influence an agent's decision. The influence of an agent that decides to evacuate in the beginning may diminish over time as the situation is changing dynamically. Therefore, instead of observing all of it's neighbors' states, an agent may only consider those who have evacuated within certain number previous time steps. We want to investigate the role of this phenomenon on the propagation of evacuation decision and on the size of the cascade.

e) An agent can have different levels of relationship with its neighbors. For instance, a relative might have higher influence on agent's decision than a friend might have. We investigate the role of this strength of ties on social contagion process. Specifically, does higher proportions of strong ties result into larger cascade size?

3. Analytical Formulation of the Social Contagion Model

In this section we determine the condition when a cascade or contagion will occur within a population of agents having a hypothetical social network. The social network has a particular mixing pattern based on community characteristics such as race, ethnicity, income etc. The analytical derivation of the cascade condition uses the key results of Newman's study (Newman, 2003b) that determines different characteristics of a network with multiple communities. We also use the threshold model proposed by Watts (Watts, 2002). However, Watts' model does not consider any community structures

Table 1: List of Notations

Notation	Description
k	Degree of a vertex
z	Average degree
$z^{(i)}$	Average degree of agents of type i
$p_k^{(i)}$	Probability that a randomly chosen vertex of type i has degree k
e_{ij}	The fraction of edges in a network that connect a vertex of type i to a vertex of type j
ϕ	Threshold value of an agent
$G_0(x)$	Generating function for the degree distribution p_k
$G_0^{(i)}(x)$	Generating function for the degree distribution $p_k^{(i)}$ for agent of type i
ρ_k	Given that an agent has degree k, the probability that it has a threshold ϕ such that $\phi \leq \frac{1}{k}$
$F_0^{(i)}(x)$	Generating function for the degree distribution of the evacuated agent of type i following a randomly chosen agent
$F_1^{(i)}(x)$	Generating function for the degree distribution of the evacuated agent of type i following a randomly chosen edge
$H_0^{(i)}(x)$	Generating function for the number of evacuated agents that can be reached from a randomly chosen agent of type i
$H_1^{(i)}(x)$	Generating function for the number of evacuated agents that can be reached from a randomly chosen edge that is connected to an agent of type i

with the network. The model developed here is a physics-based model and abstracts the key variables which influence the propagation of evacuation decisions in a social network. The purpose of this section to introduce some results reported in the network science literature.

Consider a mixing matrix e_{ij}, degree distribution of agents $p_k^{(i)}$ of type $i = 1, 2, ..., n$ and a distribution of threshold values of $f(\phi)$. Assume that an agent of type i has degree k (i.e. it has k neighbors) and the relations between this agent and its other neighbors are referred as edges. These k edges of the agent are divided into n types (where each type corresponds to the community they belong) with some partition $\{r_1, r_2, ..., r_n\}$ where $\sum_{j=1}^{n} r_j = k$.

Now, the probability that a partition $\{r_j\}$ takes a particular value is given by the following equation of multinomial probability (Newman, 2003b)

$$P^i(k, \{r_j\}) = k! \prod_j \frac{1}{r_j!} \left[\frac{e_{ij}}{\sum_j e_{ij}} \right]^{r_j} \tag{1}$$

Generating function (Wilf, 1994) is frequently utilized in network science to derive different properties of distributions. For instance, to derive the probability distribution of node degrees k of a graph, the corresponding generating function $G_0(x)$ can be stated by the following equation:

$$G_0(x) = \sum_{k=0}^{\infty} p_k x^k \tag{2}$$

where p_k is the probability that a randomly chosen vertex on the graph has degree k. The distribution p_k has to be correctly normalized so that

$$G_0(1) = 1 \tag{3}$$

The probability generating function, for example $G_0(x)$, has a number of properties that are useful in deriving important relations in network science. For a detailed review of the properties readers are referred to the reference Newman et al. (2001).

The generating function for the distribution of the number of edges for each type can be written as (Newman, 2003b):

$$
\begin{aligned}
G_0^{(i)}(x_1, x_2, ..., x_n) &= \sum_{k=0}^{\infty} p_k^{(i)} \sum_{\{r_j\}} \delta(k, \sum_j r_j) \times P^i(k, \{r_j\}) x_1^{r_1} x_2^{r_2} ... x_n^{r_n} \\
&= \sum_{k=0}^{\infty} p_k^{(i)} \left[\frac{\sum_j e_{ij} x_j}{\sum_j e_{ij}} \right]^k \\
&= G_0^{(i)} \left(\frac{\sum_j e_{ij} x_j}{\sum_j e_{ij}} \right) \qquad (4)
\end{aligned}
$$

Where, δ is the Kronecker delta function and $G_0^{(i)}(x) = \sum_k p_k^{(i)} x^k$ is the generating function for the degree distribution $p_k^{(i)}$.

Now, one important feature to find is the distribution of the degree of agent of type i following a randomly chosen edge,

$$
G_1^{(i)}(x) = \frac{\sum_k k p_k^{(i)} x^{k-1}}{\sum_k k p_k^{(i)}} = \frac{1}{z_i} G_0^{(i)'}(x) \qquad (5)
$$

Where $z_i \equiv G_0^{(i)'}(1)$ is the mean degree of type i agents.

For the distribution of the edges of different types (Newman, 2003b),

$$
G_1^{(i)}(x_1, x_2, ..., x_n) = G_1^{(i)} \left(\frac{\sum_j e_{ij} x_j}{\sum_j e_{ij}} \right) \qquad (6)
$$

Now consider a threshold model where a randomly chosen single agent decides to evacuate first; this agent is referred as a *seed agent*. The evacuation pattern from this seed agent will propagate if at least one of its neighbors has degree k and threshold ϕ such that $\phi \leq \frac{1}{k}$ and the corresponding probability is $\rho_k = P[\phi \leq \frac{1}{k}]$ (Watts, 2002). We call this agent as *evacuated* agent. Thus the probability of any agent u having degree k and having evacuated is $\rho_k p_k$ and the corresponding generating function for evacuated agent degree will be $F_0(x) = \sum_k \rho_k p_k x^k$ (Watts, 2002). Similarly, for our case, the generating function for evacuated agent degree of type i following a randomly chosen agent will be

$$
F_0^{(i)}(x) = \sum_k \rho_k^{(i)} p_k^{(i)} x^k \qquad (7)
$$

Two important quantities can be found from Eq. 7. They are: a) the fraction of evacuated agents of type i, $P_v^{(i)} = F_0^{(i)}(1) = \sum_k \rho_k^{(i)} p_k^{(i)}$, and b) the average degree of an evacuated agent of type i, $z_v^{(i)} = F_0^{(i)'}(1) = \sum_k \rho_k^{(i)} k p_k^{(i)}$.

Similar to Eq. 4, the generating function for the distribution of the number of evacuated edges connecting to each type of agent is: $F_0^{(i)}(x_1, x_2, ..., x_n) = F_0^{(i)}\left(\frac{\sum_j e_{ij} x_j}{\sum_j e_{ij}}\right)$ and similar to Eq. 5 the generating function for the distributions of the evacuated agent degree of type i following a randomly chosen edge, $F_1^{(i)}(x) = \frac{1}{z^{(i)}} F_0^{(i)'}(x)$. For the distribution of the edges of different types, $F_1^{(i)}(x_1, x_2, ..., x_n) = F_1^{(i)}\left(\frac{\sum_j e_{ij} x_j}{\sum_j e_{ij}}\right)$

An important distribution to observe is the distribution of the number of evacuated agents that can be reached from a randomly chosen agent of type i. The generating function $H_0^{(i)}$ for such distribution satisfies the following self-consistency condition (Newman, 2003b)

$$H_0^{(i)}(x) = 1 - F_0^{(i)}(1) + x F_0^{(i)}\left[H_1^{(1)}(x), ..., H_1^{(n)}(x)\right] \tag{8}$$

And similarly the distribution of the number of evacuated agents that can be reached by following a random edge connected to an agent of type i can also be found (Newman, 2003b):

$$H_1^{(i)}(x) = 1 - F_1^{(i)}(1) + x F_1^{(i)}\left[H_1^{(1)}(x), ..., H_1^{(n)}(x)\right] \tag{9}$$

The average number of evacuated agents $s_v^{(i)}$ reachable from an agent of type i is

$$\begin{aligned}
s_v^{(i)} &= \left.\frac{dH_0^{(i)}}{dx}\right|_{x=1} \\
&= F_0^{(i)}\left[H_1^{(1)}(1), ..., H_1^{(n)}(1)\right] + F_0^{(i)'}(1)\frac{\sum_j e_{ij} H_1^{(j)'}(1)}{\sum_j e_{ij}} \\
&= P_v^{(i)} + F_0^{(i)'}(1)\frac{\sum_j e_{ij} H_1^{(j)'}(1)}{\sum_j e_{ij}} \\
&= P_v^{(i)} + z_v^{(i)}\frac{\sum_j e_{ij} H_1^{(j)'}(1)}{\sum_j e_{ij}}
\end{aligned}$$

This can be written in matrix format,

$$\mathbf{s_v} = \mathbf{P_v} + \mathbf{m_0}\mathbf{H_1'}(\mathbf{1}) \tag{10}$$

Where $\mathbf{m_0}$ is a matrix $[m_0]_{ij} = \frac{z_v^{(i)} e_{ij}}{\sum_j e_{ij}}$

Now to find $\mathbf{s_v}$, we will need to know $\mathbf{H_1^{'}(1)}$

$$\begin{aligned} H_1^{(i)'}(1) &= F_1^{(i)}\left[H_1^{(1)}(1), ..., H_1^{(n)}(1)\right] + F_1^{(i)'}\left[H_1^{(1)}(1), ..., H_1^{(n)}(1)\right] \\ &= F_1^{(i)}(1) + F_1^{(i)'}(1)\frac{\sum_j e_{ij} H_1^{(j)'}(1)}{\sum_j e_{ij}} \end{aligned}$$

In matrix format,

$$\mathbf{H_1^{'}(1)} = \mathbf{F_1(1)} + \mathbf{m_1 H_1^{'}(1)} = \mathbf{F_1(1)[I - m_1]^{-1}} \tag{11}$$

Where $\mathbf{m_1}$ is a matrix $[m_1]_{ij} = \frac{F_1^{(i)'}(1) e_{ij}}{\sum_j e_{ij}}$ and

$$F_1^{(i)'}(1) = \frac{1}{z^{(i)}} F_0^{(i)''}(1) = \frac{1}{z^{(i)}} \sum_k \rho_k^{(i)} p_k^{(i)} k(k-1) = \frac{1}{z^{(i)}}(\langle k^2 \rangle - \langle k \rangle) = \frac{z_{v2}^{(i)}}{z_1^{(i)}}$$

Where $z_1^{(i)} \equiv z^{(i)}$ (i.e. average degree of agents of type i) and $z_{v2}^{(i)}$ is the average second evacuated neighbors of an agent of type i.

So matrix $\mathbf{m_1}$ becomes, $[m_1]_{ij} = \frac{z_{v2}^{(i)} e_{ij}}{z_1^{(i)} \sum_j e_{ij}}$

So finally, Eq. 10 for average size of the cluster of evacuated agents becomes,

$$\mathbf{s_v} = \mathbf{P_v} + \mathbf{m_0}\left[\mathbf{I} - \mathbf{m_1}\right]^{-1}\mathbf{F_1(1)} \tag{12}$$

A cascade will happen when s_v diverges and s_v will diverge when $det[\mathbf{I} - \mathbf{m_1}]$ will reach its first zero. It is also important to characterize when the cascade occurs in a social network with multiple communities.

$$\begin{aligned} \frac{z_{v2}^{(i)} \sum_j e_{ij}}{z_1^{(i)} \sum_j e_{ij}} &= 1 \\ \Rightarrow z_{v2}^{(i)} &= z_1^{(i)} \end{aligned} \tag{13}$$

This means that a cascade will happen when the average *first neighbors* of type i are equal to the average second evacuated neighbors of that same type. This cascading condition can be interpreted as, when $z_{v2}^{(i)} < z_1^{(i)}$ the initial evacuated cluster of type i is too small to generate a cascade within its own community and hence the whole network. However when $z_{v2}^{(i)} > z_1^{(i)}$ the

typical size of the evacuated clusters of type i is large enough for the cascade to percolate through their own community. However, for the cascade to propagate across all the communities, each of the communities will have to satisfy the above condition.

4. Simulation Model

In this work, we develop a simulation model to investigate the performance of the analytical findings as compared to simulations on many random social networks. Briefly, the simulation model consists of two stages: first stage builds a social network with multiple communities with a specific mixing pattern. This stage follows the algorithm proposed by Newman (Newman, 2003b) for creating networks with multiple communities. The second stage applies the threshold model of social contagion on the generated network. The essential steps to setup the simulation model are as following:

Stage I - Building the mixed social network

1. Choose the size of the network in terms of the number of edges $M$and then draw M edges from the distribution e_{ij}. This step generates M edges that are identified by the types of the vertices they connect. These M edges are created in such a way that the proportion of the edges between vertices of type i and j will be e_{ij} as M becomes large.

2. Count the number of ends of edges of each type i, to obtain the sum of m_i; calculate $n_i = \frac{m_i}{z_i}$ where z_i is the desired mean degree of agents of type i.

3. Draw n_i agents from degree distribution $p_k^{(i)}$ of agent type i making the sum of degrees of the agents to be m_i. Notice that in general the sum of the vertex degrees will not be equal to m_i. Newman (2003b) suggests to select and discard one vertex randomly, and draw another vertex from $p_k^{(i)}$ until the sum of the degrees becomes equal to m_i. Multiple draws are required for the vertices to match m_i and different combinations should be tried until all nodes have the required degree which is a cumbersome process.

4. Randomly pair up the m_i ends of edges of type i with the generated agents so that each agent has the number of attached edges according to its chosen degree.

5. Repeat step 3 and 4 for each agent type.

Stage II - Applying the threshold model

1. Each agent is given a fixed threshold value, ϕ that is sampled from $f(\phi)$.

2. There are two possible states σ_0 (Not evacuated) and σ_1 (Decided to evacuate) for each agent. Neighbors of the agents who have evacuated update their states at times $t = 0, 1, 2, ...$

3. At a given time step each neighbor of the evacuated agents observes the fraction of its neighbors in state σ_1 and switches to σ_1 if the fraction exceeds its threshold ϕ.

The simulation model is coded in C using the iGraph library (igraph, 2006).

5. Results and Discussion

Our model is applicable to any population of agents forming a network with arbitrary degree distributions $p_k^{(i)}$ for each agent type i and threshold distributions $f(\phi)$. However, here we demonstrate the features of our model assuming a uniform degree distribution (i.e. number of neighbors of each vertex is uniformly distributed between k_1 and k_2) with a particular average degree (z) for each type of agent. Here we consider the communities as neighborhoods located in a region and the degree of an agent as the size of the core network (i.e. people with whom one discusses important matters) of the agent. This type of degree distribution is a reasonable assumption as Hampton et al. (2011) observed such distribution of the core network size for individuals. We also assume a uniform threshold distribution between ϕ_1 and ϕ_2. To make the network and the corresponding analysis simple we here assume a network with two communities with the following edge distribution matrix:

$$\begin{pmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{pmatrix}$$

The communities are also assumed to have same degree distribution, however in a later experiment we relax this assumption. To investigate the effect of the assumed degree distribution on our results we also consider Poisson degree distribution; the relevant results related to the Poisson degree distribution are given in the Appendix. For each of the setup we run the simulation for 100 realizations. Whenever appropriate, simulation results are averaged over these 100 realizations. Each realization builds a random network of 2,000 edges in which a single agent makes the decision to evacuate at time $t = 0$.

In this section we present the following computational results:

a) Comparison between the cascading condition obtained from the analytical model and that obtained from the simulation model

b) Effect of the mixing pattern on cascade propagation

c) Effect of the initial seed on cascade propagation for uniform degree distribution

d) Effect of the previous time-steps considered by the agents on average cascade size

e) Effect of the strength of ties on average cascade size

5.1. When does a cascade occur?

Fig. 1 expresses the cascade condition (Eq. 13) graphically as a boundary of average degree and average threshold value (z, ϕ). The solid line corresponds to the analytical solution and the dotted line represents the the result obtained from the simulation model. The cascade condition obtained from the analytical model matches closely with the simulation model. The area where no global cascade occurs represents the situation when $z_{v2} < z_1$ i.e. the cluster of evacuated agents generated from the initial seed agent is too small to generate a cascade within its own community. On the other hand, the area where global cascade occurs represents the situation when $z_{v2} > z_1$ i.e. the size of the cluster of evacuated agents generated from the seed agent is large enough for the cascade to propagate through their own community.

An important quantity to observe is the change of average cascade size (i.e. average fraction of the evacuated agents to the total number of agents) with average threshold values and average degree of the communities. Fig. 2(a) and Fig. 2(b) illustrate how average cascade size is influenced by the average threshold values for different average degree of communities. Fig. 2(a) suggests that when average degree of a particular community is very low the average cascade size may not attain the full network even for very low threshold values.

These findings have important implications in terms of evacuation planning and management. For example, for successful evacuation additional interventions may be needed to influence individuals in communities with low average degree (i.e. community of individuals having few peers). On the other hand, mass evacuations are expected to occur in communities with high average degree (i.e. community of individuals having many peers).

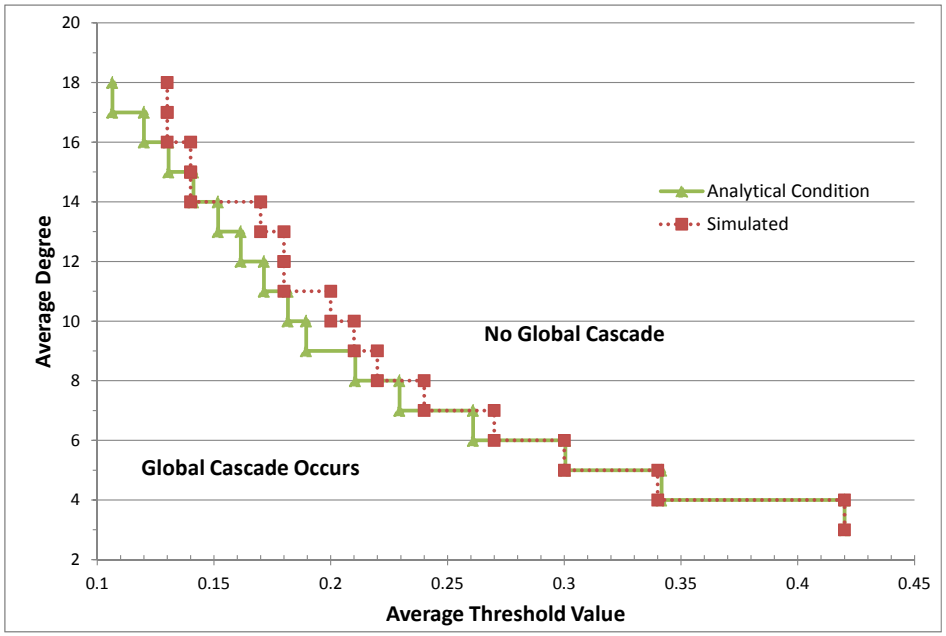


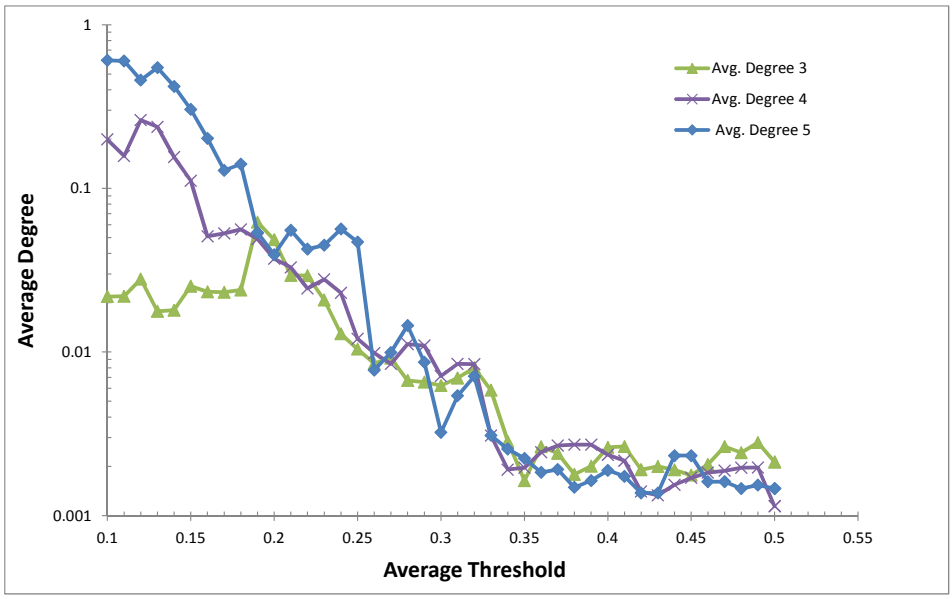
Figure 1: The condition for a cascade to occur

5.2. Role of mixing pattern on cascade propagation

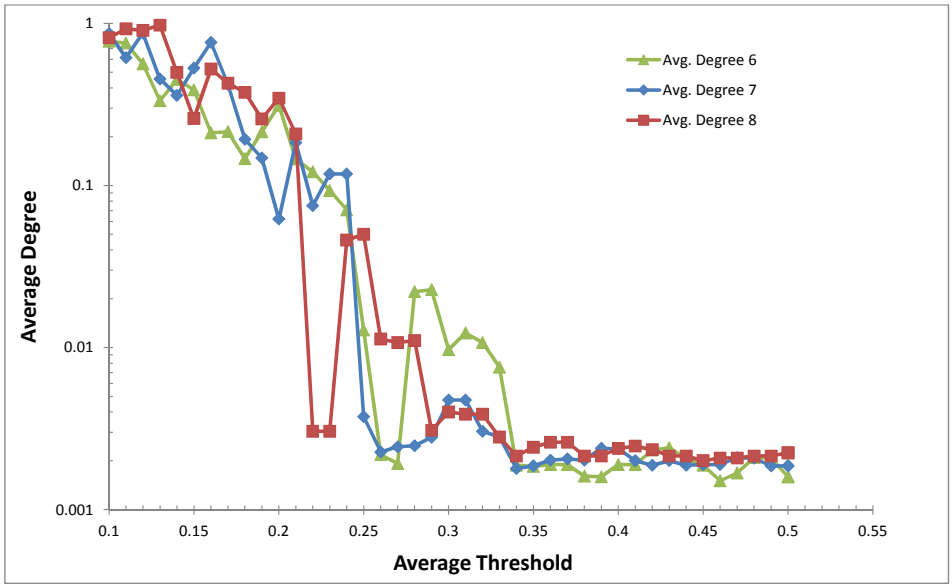
In previous section we investigated the cascading condition in a network with a particular edge distribution matrix. Here we investigate the effects of the mixing patterns on cascade propagation by changing the mixing patterns over simulation runs. We change the fraction of the intra-community and inter-community edges as specified by the matrix mentioned in Section 5.1. Our goal here is to identify how the interaction among the communities are influencing the cascade propagation in the population as a whole. Does the cascade of evacuation decision propagate faster if people have more inter-community connections?

Two kinds of approach for changing the fraction of edges are followed: a) keep the edge matrix symmetric (i.e. both communities will have the same level of intra-community edges) b) keep the edge matrix asymmetric (i.e. both communities will have different level of intra-community edges). The quantity that we observe is the average fraction of the evacuated agents along the simulation time line.

Fig. 3(a) and Fig. 3(b) illustrate the role of mixing patterns on cascade

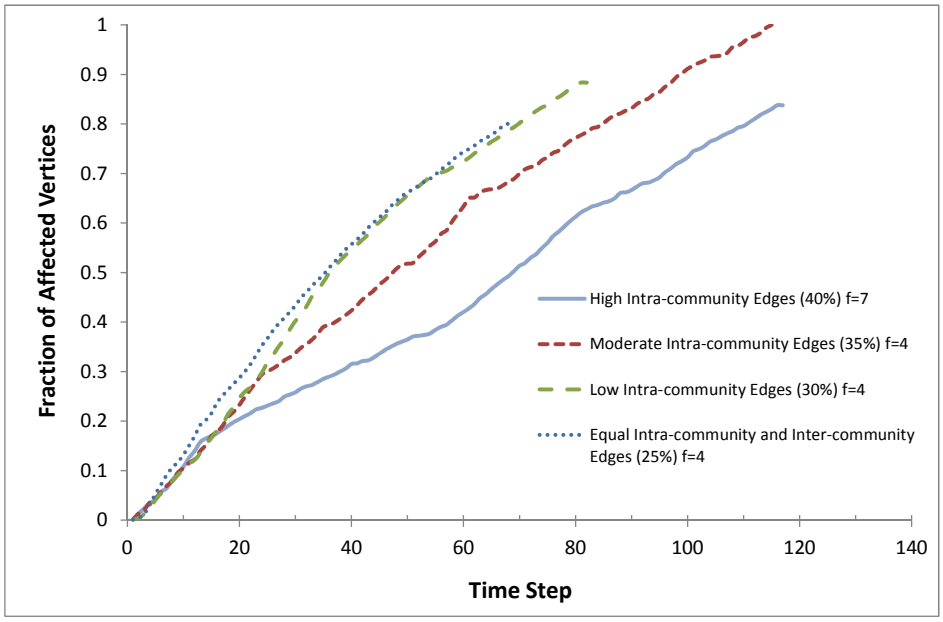


(a)

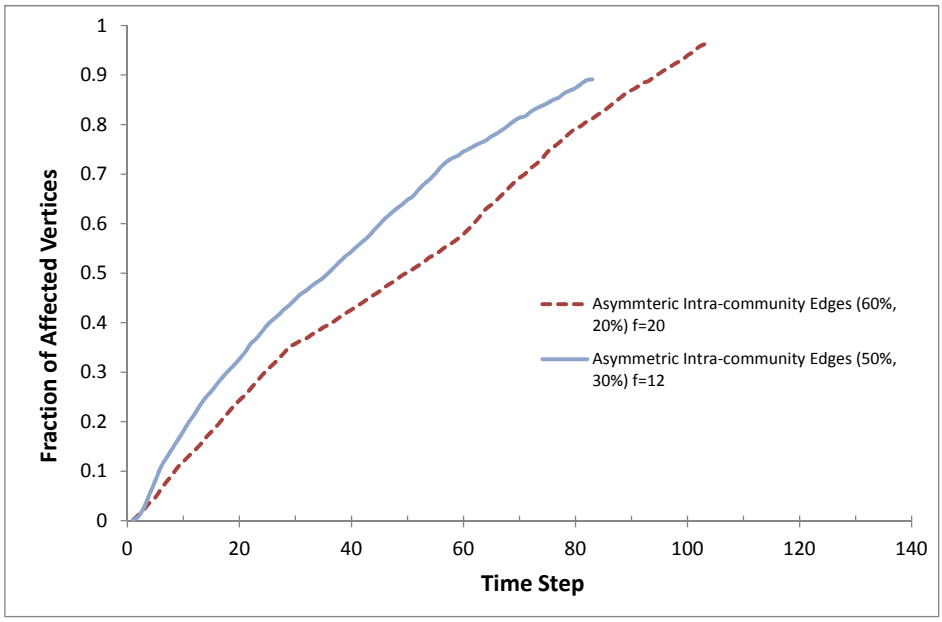


(b)

Figure 2: Average cascade size vs. average threshold values for different average degree of communities



(a) Cascade propagation for symmetric edge distribution (Average degree=6 and average threshold=0.15)



(b) Cascade propagation for asymmetric edge distribution (Average degree=6 and average threshold=0.1)

Figure 3: Role of mixing patterns on cascade propagation (Note: Due to the randomness in the simulation, different simulation may end at different time steps. Thus it is not possible to take the average of the 100 realizations. We first find the frequency distribution of simulation end times for those realizations in which at least 80% of the vertices decide to evacaute. We then report the average of the results for those realizations that have the highest frequency of simulation end time where f= frequency of the simulation end time)

propagation for symmetric and asymmetric edge distributions respectively. Fig. 3(a) suggests that as we decrease the intra-community edges which is equivalent to increasing inter-community edges, cascade propagates faster. This means that the more the inter-community edges are the faster the propagation of the cascade is. These inter-community edges can serve the purpose of bridges and let the information or the influence to propagate faster. The propagation curve has different stages, the initial stage consists of the propagation of cascade within the community of the initial seed and in the subsequent phases it starts to propagate to the other community. When the proportion of intra-community and inter-community edges are equal the whole network acts like a single community and in this case the rate of propagation is the highest. Given that communities are common in our societies, this analysis shows the benefits of having more connections among communities in the social network regarding the contagion process.

Fig. 3(b) presents the propagation of cascade when the communities are not equally dense. That is they have different proportion of edges among themselves. Similar to the previous finding, it demonstrates that the cascade propagates faster in population with less difference in the intra-community edges. That is it indicates that as the communities become more similar to one another the faster the cascade propagate among the population.

These findings can be translated into important implications for managing hurricane evacuations. For example, population with greater level of inter-community edges are expected to respond faster to evacuation notices or warning compared to population with lower level of inter-community edges. This suggests that intervention strategies are to be implemented for the latter communities.

5.3. Role of initial seed

It is important to note that we initialize our simulation by switching the state of a single agent. That is at time $t = 0$ a single agent decides to evacuate. This agent is acting as an initial seed for our contagion simulation process. A natural question therefore would be understand the role of this initial seed on cascade propagation. In this section, we investigate the role of this initial seed on cascade propagation by experimenting with two types of initial seed. One obvious candidate for such seed might be the agent with the highest degree. We compare the rate of propagation of cascade initiated from a random seed with that initiated from a seed which has the highest degree. Fig. 4 presents the cascade propagations for random seed

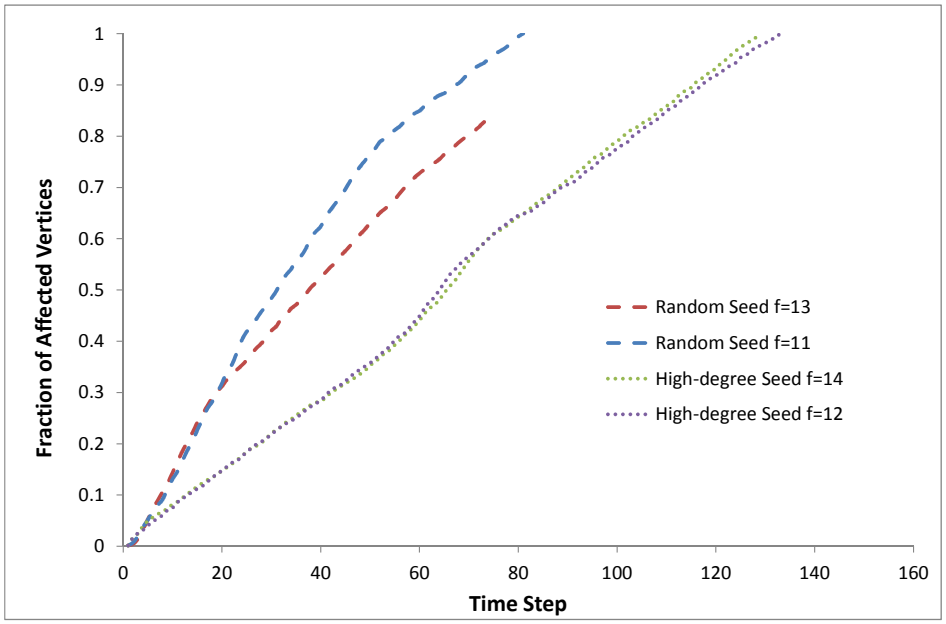


Figure 4: Role of initial seed on cascade propagation (Average threshold=0.1 and f= frequency of the simulation end time)

and highest degree seed for uniform degree distribution. Interestingly we find that the cascade initiated from a random seed propagates faster than that initiated from the highest degree seed. One plausible explanation for this might be the cascading condition described in Eqn. 13. When we select the highest degree seed the local neighborhood of that initial seed is relatively larger than the affected cluster size (i.e. $z_1 > z_{v2}$); this makes the affected cluster to propagate slowly. On the other hand, if we select a random seed then the expected degree of the seed will be the average degree of the degree distribution. Therefore the local neighborhood will be relatively smaller than the affected cluster size (i.e. $z_1 < z_{v2}$) which makes the cascade to propagate fast.

5.4. Role of previous time steps considered by the agents

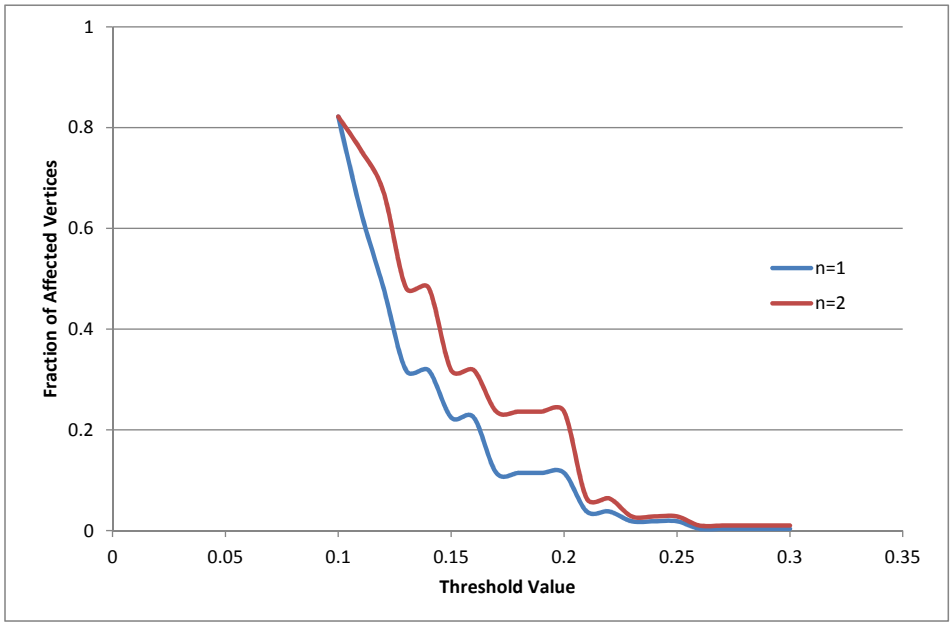
In our simulation model (see stage II of the model described in Section 4), every neighbor of the evacuated agents observes the fraction of its neighbors in state σ_1 (i.e. evacuated) and switches to σ_1 (i.e. decides to evacuate) if the fraction exceeds its threshold value. Here we run experiments on previous time steps based on which agents will make their decisions. That is an agent

will count only those who have evacuated in previous n time steps. Those who have evacuated before n time steps do not have any influence on agent's evacuation decision. Analyzing the results of the the experiments for n=1, 2, 3 and 4, we observe no specific trend in the difference of the cascade propagation rate. However, we observe difference in the size of the cascade and simulation end times (the time to reach a stable situation) but only between the cases for n=1 and n=2. Beyond n=2 there are no differences in the average size of the cascades and the time to stabilize.

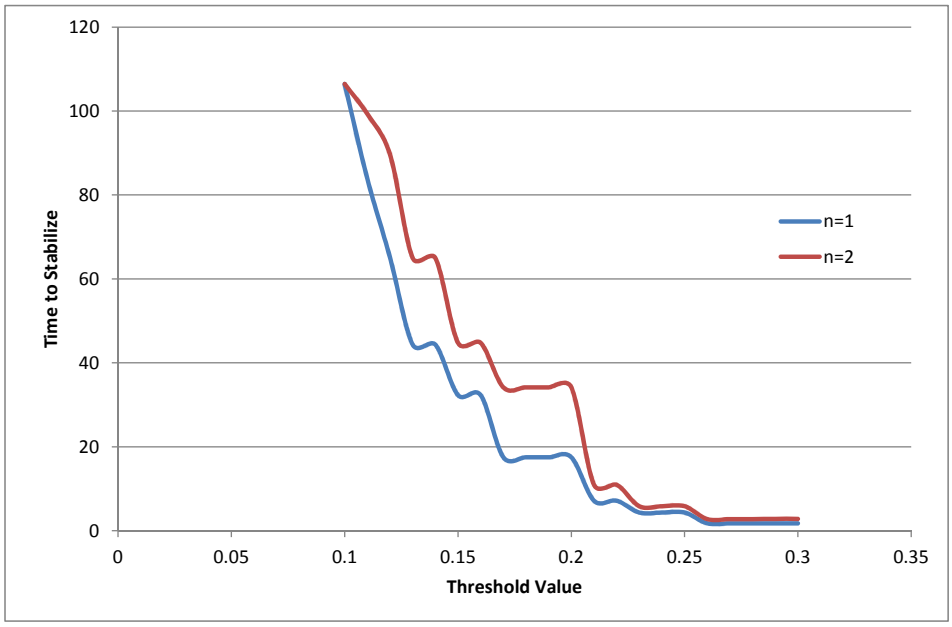
Fig. 5(a) and Fig. 5(b) present the role of previous time-steps considered on average cascade size and the time to stabilize respectively. This analysis demonstrates the sensitivity of time-steps on the findings of our model. It shows that cascade size is not significantly different for different time horizons. However, cascade size is relatively higher if agents count the number of neighbors who evacuated in last two time steps compared to the case when they only count the number of neighbors who evacuated only in the previous time step. This is plausible as the fraction of neighbors who evacuated is greater if agents consider more previous time steps. In other words if people consider only those who evacuated recently then it is less likely to have a global cascade of significant size than the situation when people consider all the previous times.

5.5. Role of the strength of ties

In previous analyses agents consider only the number of its neighbors for making the evacuation decision. However, it likely that different relationships will have different level of influence on individual's decisions. For instance, a member of the agent's kin network will have higher level of influence than that of a member of its nonkin network. For analyzing the effects of the strength of relationships we assign each tie a weight using two categories of ties (i.e. string tie and weak tie). A strong tie (weight=3) represents the relationship with a kin (e.g. spouse, parent, sibling, child, or other family member) and a weak tie (weight=1) represents the relationship with nonkin (e.g. co-worker, friend, advisor, neighbor, or group member). Simulations are run by changing the overall proportions of the strong and weak ties. Instead of calculating the fraction of the affected neighbors, agents now calculate the weighted fraction of the affected neighbors (i.e. total weights of the ties connected to the affected neighbors/total weights of the ties connected to all neighbors) to compare against its threshold value.



(a) Average cascade size vs. threshold value for different previous times-steps considered(Average degree=6)



(b) Time to stabilize vs. threshold value for different previous times-steps considered(Average degree=6)

Figure 5: Role of previous time-steps considered by the agents. Agents count only those who evacuated in previous n steps

Fig. 6 presents the role of the strength of ties on average cascade size. In general, we observe that until a threshold value 0.15, there is a direct relationship between the strength of ties and the average cascade size. That is stronger the ties, greater is the average cascade size. This result is contrary to finding of "the strength of weak ties" by Granovetter (1973). However, weak ties may provide greater cascade size for simple contagion but when weights are introduced in the network the strength of weak tie is no longer found. Since agents assign higher values to the strong ties, the likelihood of cascade propagation increases with the proportion of increase in strong ties. This indicates that when the network has higher proportion of strong ties (e.g. discussion with kin about evacuation decision) the size of the evacuated agents will be larger compared to the network with smaller proportion of strong ties.

This finding has important implication about the role of social media. For instance, with the advent of so called online social media sites (e.g. Facebook, Twitter etc.), one can imagine sending the hurricane warning messages through these media and expecting greater cascade size. However, our analysis reveals that this is very unlikely. We observe that the decision to evacuate is not only strongly influenced by whom an agent is "linked" to but also the trust (weight) that the agent places on the connected nodes.

6. Summary and Conclusions

In this paper we investigate the effect of social connections on the contagion process of decision making. It is found that the growing literature on network science can provide an appropriate tool for investigating this phenomenon. The methodology for introducing social influence is demonstrated here. We model the social contagion process in a population with multiple communities and investigate the effect of community characteristics on the contagion process. A simple threshold model of decision making is applied to a hypothetical social network. Specific questions related to the social contagion process are answered: a) when does a cascade occur? b) what is the role of mixing patterns on cascade propagation? c) how to control the propagation of cascade? d) what is role of previous time steps on threshold model? e) what is the role of the strength of ties on the contagion process?

We derive an analytical condition for any cascade to happen in the network with a particular mixing distribution among communities. Our simulation model gives values of the cascading condition close to the ones obtained

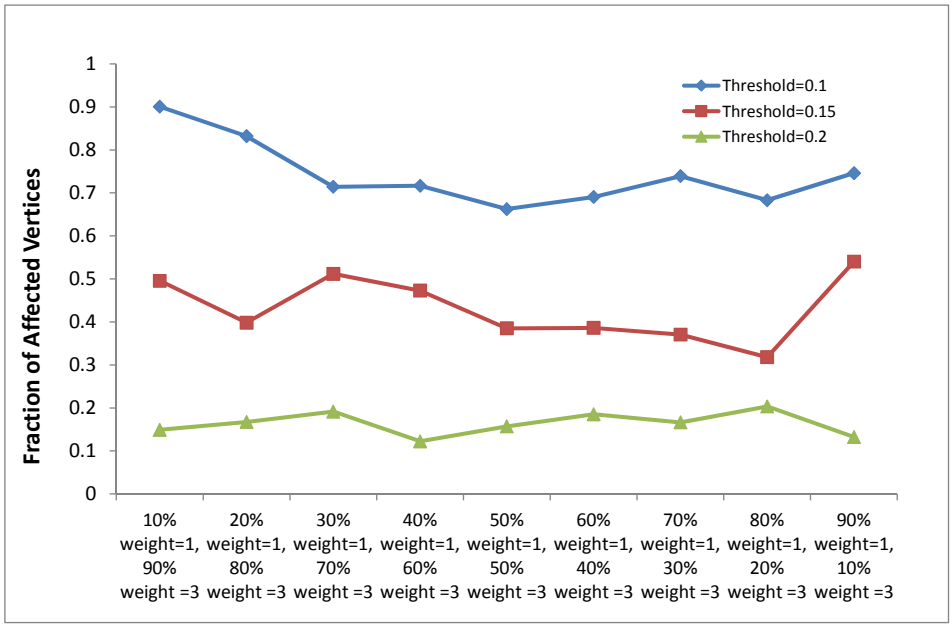


Figure 6: Average cascade size vs. different distributions of the strength of ties (Average degree=6)

from the analytical model. We also identify the role of mixing patterns on cascade propagation. It is found that as inter-community edges increase in the network the cascade propagates faster. On the issue of how to control the propagation of the cascade we found that a high degree seed does not necessarily propagate the cascade faster than a cascade initiated by a randomly selected seed.

In this paper, we also demonstrate how to relate the results from our model to hurricane evacuation. Previous literature indicated the existence of social influence on household's hurricane evacuation decision making. We present a methodology to characterize such social influence. The important insights on social influences have implications to evacuation management as well as policy making. We highlight several specific implications.

First, as suggested from our results that mixing patterns influence the cascade propagation, evacuation manager can take into account the existing social network structure to anticipate the propagation of a cascade. For example, if there exists a community with very few average inter-community edges extra actions might be taken to influence the propagation of information (i.e. evacuation warning). The communities in our analysis can be

interpreted as neighborhoods present in geographic sense. In this case, our findings also suggest land use designs favoring such cascading condition.

Second, using this type of analysis we can compare different strategies to expedite the propagation of information (i.e. hurricane warning) in a network. For example, in our analysis we found that faster propagation of warning can be achieved if agents are selected randomly to have the warning first than selecting individuals with greater number of neighbors. Although this type of strategy may not be practical in actual hurricane evacuation context, the model presented here is general enough to develop and evaluate other types of strategies favoring faster propagation of hurricane warning.

Third, results indicate that the proportions of strong ties in the network influence the average size of the evacuated agents. When agents make decisions based on the level of weights of the relationship with the evacuated neighbors, higher proportion of strong ties result into greater cascade size. Evacuation managers can expect higher level of compliance or non-compliance behavior if the underlying social network has higher proportion of strong ties.

This analysis can also contribute to the transportation research area in terms of building a modeling framework which incorporates the social influence on decision making relevant for transportation modeling. Future research can address several aspects of the evacuation choice problem considering the social influence. For instance, dynamic evacuation choice models, specifying a functional form of the threshold values of the agents, can be estimated. Thus in addition to the social influence, such kind of models will be able to estimate the influence of the individual socio-economic variables. Another avenue of research might be to find appropriate community structure or warning strategies favoring faster propagation of evacuation decisions. Although the social contagion models have been studied in network science literature more commonly, they have not been used in any previous transportation literature to the best of our knowledge. We believe that such introduction of social influences in evacuation behavior analysis can improve the modeling capabilities and can become an essential component for future integrated transportation models.

7. Acknowledgments

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ing and Dynamic Transportation Modeling in Hurricane Evacuation: An Integrated Social Science-Engineering Approach" for which the authors are grateful. However, the authors are solely responsible for the findings of the research work.

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APPENDIX

Appendix A. Experiments with Poisson degree distributions

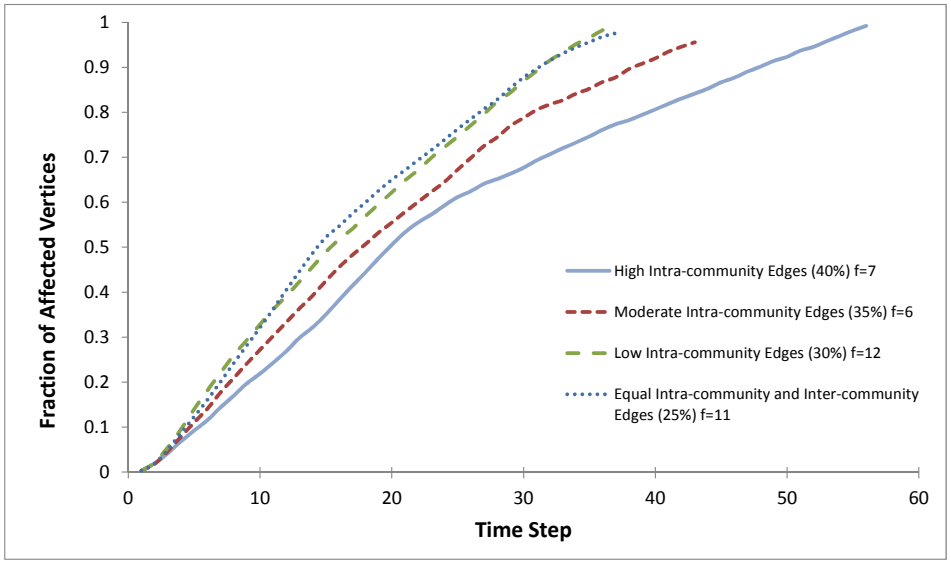


Figure A.7: Role of mixing patterns on cascade propagation for Poisson degree distribution(Note: Due to the randomness in the simulation, different simulation may end at different time steps. Thus it is not possible to take the average of the 100 realizations. We first find the frequency distribution of simulation end times for those realizations in which at least 80% of the vertices decide to evacaute. We then report the average of the results for those realizations that have the highest frequency of simulation end time where $f=$ frequency of the simulation end time)

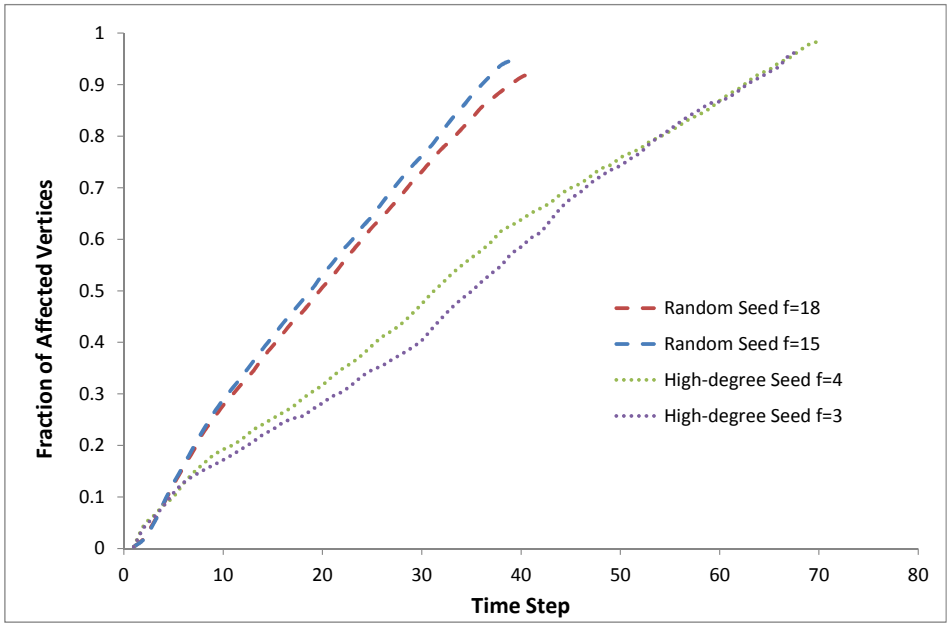
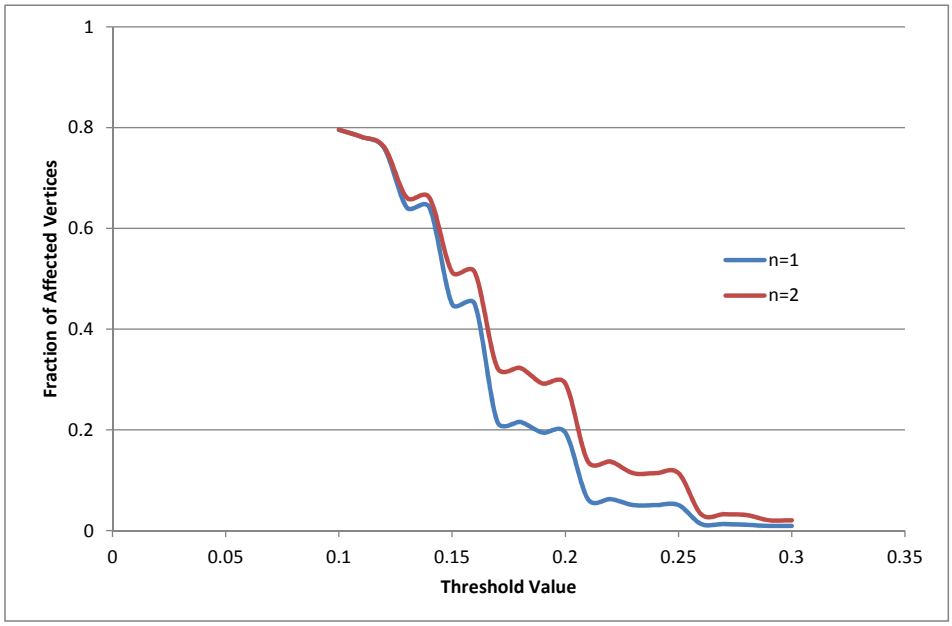
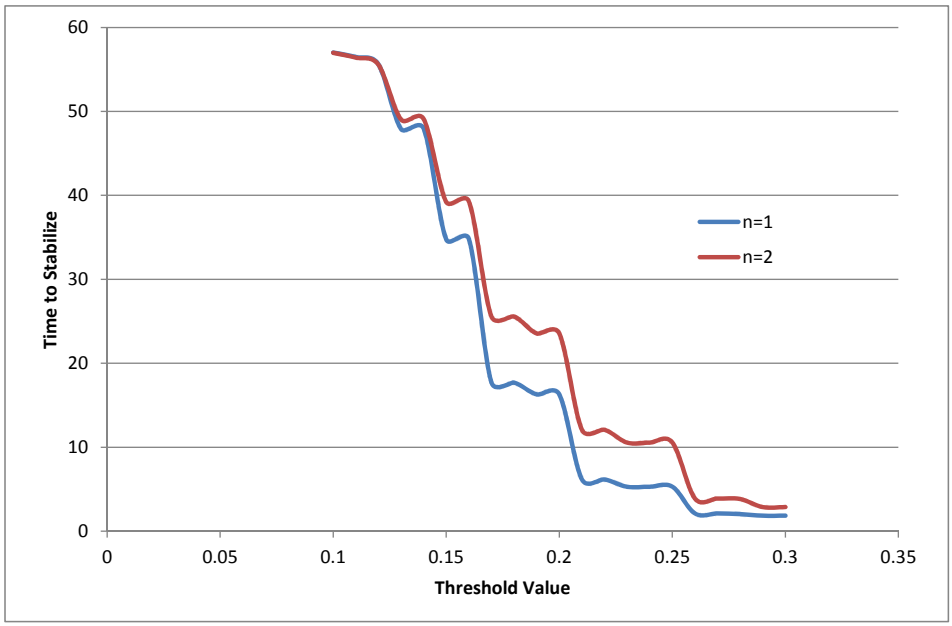


Figure A.8: The role of initial seed on cascade propagation for Poisosn degree distribution (f= frequency of the simulation end time)



(a) Average cascade size vs. threshold value for different previous times-steps considered



(b) Time to stabilize vs. threshold value for different previous times-steps considered

Figure A.9: Role of previous time-steps considered by the agents for Poisson degree distribution. Agents count only those who evacuated in previous n steps