Lecture 9: An Equilibrium Business Cycle Model
See Barro Ch. 8

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We built a model of GDP growth (trend)

Production function, TFP growth

We also built in consumption, labor, capital markets

Can talk about how the system responds to shocks in the short and long run
Now let’s try to use it to understand business cycles

A medium-term negative shock to TFP, “A”, causes:

- Wages to go down (lower productivity)
- Interest rates to go down (lower productivity)
- Investment to go down a lot (lower interest rates and too high capital stock)
- Consumption to go down a little (people like to smooth and eat out of savings (capital stock))

If all these move together in the business cycle, we have a candidate for the cause of the business cycle
Trend and Deviation

First we have to separate the “trend” from the “business cycle” or “deviation.”

The basic idea:

Real GDP = Real GDP Trend + Real GDP Deviations

How do we split them?

Define a trend and subtract the difference to find deviations:

Real GDP Deviations = Real GDP − Real GDP Trend

Note: I’ll do everything in per-capita terms, but everything looks like Barro

Note: Barro uses an “HP”-filter, which allows itself to change slope slowly, while I’ll use both the HP and an unchanging linear trend
GDP-Trend and Deviations-II

GDP over Time

Billions of Dollars

Quarter
GDP-Trend and Deviations-VI

Percent deviations over Time

Linear fit to log

Percent of Trend

Quarter

GDP and trends over Time

Tens of Thousands of Dollars

Quarter


- GDP
- Linear Trend
- HP Trend
GDP deviations over Time
Linear and HP fit to log

Tens of Thousands of Dollars


Quarter

Linear Trend  HP Trend
GDP-Trend and Deviations-IX

Percent deviations over Time
Linear and HP fit to log

Quarter

Percent of Trend
-10 -5 0 5 10

Linear Trend  HP Trend
GDP Trend and Deviations - X

GDP and trends over Time

Tens of Thousands of Dollars

Quarter

1800 1850 1900 1950 2000

GDP  Linear Trend  HP Trend
GDP-Trend and Deviations-XI

GDP deviations over Time
Linear and HP fit to log

Tens of Thousands of Dollars

Quarter

1800 1850 1900 1950 2000

-0.5 0 0.5 1

Linear Trend  HP Trend
GDP Trend and Deviations-XII

Percent deviations over Time
Linear and HP fit to log

-40
-20
0
20
40

1800 1850 1900 1950 2000
Quarter

-40
-20
0
20
40

Percent of Trend

Linear Trend
HP Trend
### Postwar Recessions

<table>
<thead>
<tr>
<th>Beginning</th>
<th>End</th>
<th>HP % Deviation (trough)</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 1948</td>
<td>October 1949</td>
<td>-6.1%</td>
</tr>
<tr>
<td>July 1953</td>
<td>May 1954</td>
<td>-2.76%</td>
</tr>
<tr>
<td>August 1957</td>
<td>April 1958</td>
<td>-3.90%</td>
</tr>
<tr>
<td>April 1960</td>
<td>February 1961</td>
<td>-2.68</td>
</tr>
<tr>
<td>December 1969</td>
<td>November 1970</td>
<td>-3.26%</td>
</tr>
<tr>
<td>November 1973</td>
<td>March 1975</td>
<td>-3.71%</td>
</tr>
<tr>
<td>January 1980</td>
<td>July 1980</td>
<td>-1.30%</td>
</tr>
<tr>
<td>July 1981</td>
<td>November 1982</td>
<td>-4.73%</td>
</tr>
<tr>
<td>July 1990</td>
<td>March 1991</td>
<td>-1.70%</td>
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<tr>
<td>March 2001</td>
<td>November 2001</td>
<td>-2.03%</td>
</tr>
<tr>
<td>December 2007</td>
<td>June 2009</td>
<td>-2.88%</td>
</tr>
</tbody>
</table>

Note: Differ from Barro a little. Standard deviation 1.63% of GDP.
## Postwar Recessions

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Note: Differ from Barro a little. Standard deviation 1.63% of GDP.
Takeaways

▶ We define a recession as when GDP is going down (peak to trough) *not* peak to peak!

▶ Deviations are very small compared to trend: typically between -1.7% and +1.7% of trend

▶ The difference between the linear and HP tells us that this last recession is a pretty big deal because we’re “off trend.”

▶ We can talk about business cycles now
The RBC Model: Shocks to the Economy

- Barro calls this an “equilibrium business cycle model.”
- Recall our production function:
  \[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \]
- A recession bops \( A \) (productivity, technology, knowledge) on the nose
- We’ve seen that growth is \( A_t \) going up
- We’ll think about business cycles are \( A_t \) shifting around
- From measurements of \( Y_t, K_t, \) and \( L_t \), we can back out what \( A_t \) is
- Then, in our model, we can see what would happen to \( Y_t, K_t, \) and \( L_t \) if agents were hit by a \( A_t \) shocks we measured
- This isn’t as tautological as it appears: only if agents make the same \( K, L \) choices as in the data will our model get the right predictions
Shocks to A-I

- It makes sense that technology/knowledge can go up
- It’s a bit less sensible to think knowledge is destroyed
- But recall Barro Chapter 5 or Lucas 1990: many things are in “A”
  - If the true production function had oil $O_t$ in it,
    \[ Y_t = A_t K_t^\alpha L^{1-\alpha} O_t^\beta \]
  - Then our measured $A$ would be shifting around by $O_t$ as well!
    \[ Y_t = (A_t O_t^\beta) K_t^\alpha L^{1-\alpha} \]
- This is just an example: it’s hard to write a model where they’re a big enough deal to cause a recession ($\beta$ is too small)
Shocks to A-II

- Many things could cause A to go down
  - Oil shocks
  - Trade shocks
  - Legal and political changes that change
    - Competitiveness
    - Trade
  - Weather & Natural Disaster shocks
  - War
Shocks to A and the labor market

- Remember the production function and profit function:

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \]

and

\[ \pi_t = A_t K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t \]

- So, taking FOC’s:

\[ w_t = (1 - \alpha) A_t K_t^{1-\alpha} L_t^{-\alpha} \]

- If \( A_t \) increases by 1%, \( w_t \) goes up by

- We can graph this as a function of \( L_t \), holding \( K_t \) and \( A_t \) fixed.
SHOCKS TO A AND THE LABOR MARKET

Effect of an Increase in the Technology Level on the Demand for Labor

$/hour

Hours/year

1700 1800 1900 2000 2100 2200
Thinking about shocks to A and the labor market

- Wage is the marginal product of labor
- When productivity goes up, demand for labor goes up, ceteris paribus
- But (in our model) labor is fixed
- If wages didn’t change, there would be a shortage of labor
- In order to get firms to demand the right amount of labor again, wages rise
Shocks to A and the capital market

- The production function and profit function:

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha} \]

and

\[ \pi_t = A_t K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t \]

- So, taking FOC’s:

\[ r_t = \alpha A_t K_t^{-\alpha} L_t^{1-\alpha} \]

- We can graph this as a function of \( K_t \), holding \( L_t \) and \( A_t \) fixed.
SHOCKS TO A AND THE LABOR MARKET
Thinking about shocks to A and the capital market

- The interest rate is the marginal product of capital

- When productivity goes up, demand for capital goes up, ceteris paribus

- But (in our model) capital is fixed

- If interest rate didn’t change, there would be a shortage of capital

- In order to get firms to demand the right amount of capital again, the interest rate rises
Putting things together

- Economic booms happen when $A \uparrow$

- Our model says that $A \uparrow \Rightarrow \{w \uparrow, r \uparrow\}$

- Recall the interest rate on bonds, $i = \frac{R}{P} - \delta$

- So our model says that $A \uparrow \Rightarrow \{i \uparrow\}$, too

- If interest rate didn’t change, there would be a shortage of capital

- In order to get firms to demand the right amount of capital again, the interest rate rises
Thinking about behavior: two different ways

▶ Recall the household’s real budget constraint (Barro Chapter 7):

\[
\text{Real Consumption} + \frac{1}{P} \Delta B + \Delta K = \text{Real Savings} = \frac{w}{P} L + i \left( \frac{B}{P} + K \right)
\]

▶ What happens when \( A \) increases?
Thinking about behavior: two different ways

- Recall the household’s real budget constraint (Barro Chapter 7):
  \[
  C + \frac{1}{P} \Delta B + \Delta K = \frac{w}{P} L + i \left( \frac{B}{P} + K \right)
  \]

- What happens when \( A \) increases?
  - \( A \) increases \( \frac{w}{P} \), as just seen, and \( L \) doesn’t change, so labor income increases.
  - \( A \) increases \( R \) (and therefore \( i \)), as just seen, and \( K \) doesn’t change in SR, so nonlabor income increases
  - The household is richer, so either consumption or real savings go up (or both)
Thinking about behavior: two different ways

Alternatively, you can think of things using the aggregate budget constraint (Barro Chapter 7):

\[
\begin{align*}
C_{\text{Real Consumption}} + \Delta K_{\text{Real Net Investment}} &= Y_{\text{Real GDP}} - \delta K_{\text{Depreciation}} \\
\end{align*}
\]

or, writing \( Y \) explicitly:

\[
\begin{align*}
C_{\text{Real Consumption}} + \Delta K_{\text{Real Net Investment}} &= AK^\alpha L^{1-\alpha} - \delta K_{\text{Depreciation}} \\
\end{align*}
\]

What happens when \( A \) increases?

- \( A \) increases \( Y \)
- \( \delta K \) is fixed in the short run
- The household is richer, so either consumption or real investment go up.
AN INCREASE IN A: INCOME AND SUBSTITUTION EFFECTS

- When A goes up, i goes up, and household is richer. What happens to consumption now and tomorrow?
  - Income effect: the household is richer, consumption in all periods goes up
  - Substitution effect: the interest rate is higher, so consumption today is more expensive than tomorrow: consumption today goes down, consumption tomorrow goes up

- Our prediction is ambiguous for the present (income up means $c_{\text{now}} \uparrow$, interest rate up means $c_{\text{now}} \downarrow$), and unambiguous in the long run.
Recall Chapter 7’s discussion of increases in income

- If all current and future incomes rise by the same amount, then consumption in all periods rise by that amount
  - In other words, you’ll eat all of your benefit today, because you’ll have it again tomorrow

- So if $A \uparrow$ permanently, then it’s likely $c_{\text{now}} \uparrow$
  - If $A \uparrow$ only today, then $c_{\text{now}}$ may go up or down.

For the duration of Chapter 8, we consider permanent shocks to $A$
Matching Theory with Facts-I

- We’re going to think that $A_t$ is moving $Y_t$ around (permanently)

- If that’s the case,
  - $C_t$ should be “procyclical” (move with GDP)
  - $C_t$ should be less volatile than $Y_t$ (why?)
    - People like to smooth consumption
  - $I_t$ should be “procyclical” (move with GDP)
  - $I_t$ should be more volatile than $Y_t$ (why?)
    - When $A_t \uparrow$, $r_t \uparrow$, so $C_t$ doesn’t go up by the amount $A_t$ does. Consequently, $I_t \uparrow$ by more to balance it out.
    - Note: empirical evidence suggests this intertemporal effect should be small, but in reality it seems to be big...we’ll think about why
Consumption procyclical, less volatile
InVESTMENT PROCYCLICAL, MORE VOLATILE

Figure 8.10 Cyclical Behavior of U.S. Real GDP and Investment
Other predictions?

- If $A_t$ is moving around then $w_t$ should be procyclical too because marginal product of labor increases with $A_t$

- If $A_t$ is moving around then $r_t$ should be procyclical too because marginal product of capital increases with $A_t$
Wages are procyclical
Interest rate is procyclical
Taking stock

- We want to analyze the business cycle

- We will do so by assuming that $A_t$, productivity/knowledge, is getting bopped around

- If that’s the case, then we would predict that:
  - $C_t$, $I_t$, $w_t$, and $r_t$ would all be procyclical
    - They are!
  - $C_t$ would be less volatile than GDP and $I_t$ would be more
    - They are!

- Things are looking pretty good, but it’s hard to justify just how little consumption moves compared to GDP: intertemporal substitution effect would have to be very large
Thinking about temporary shocks

So far we’ve been thinking about permanent shocks

When a shock is permanent, it hits all periods equally, so all consumptions rise by roughly the same amount

What about when a shock is temporary?
▶ Then still big effects on the interest rate
▶ But now small effects on consumption, because income effect spread over many periods

If shocks are temporary, then consumption would barely covary with GDP
Consumption procyclical, less volatile
Temporary and Permanent Shocks

- We’re faced with a conundrum
  - If shocks are permanent, then consumption and GDP move together because all periods wealthier
    - But to explain investment being so volatile, we would need an unrealistic intertemporal effect
    - Another way of putting this is that Y and C move together too much with permanent shocks
  - If shocks are temporary, then consumption and GDP don’t move together income spread over many periods
    - But it explains investment volatility well!
  - When you come to a fork in the road, take it: we split the difference and conclude that shocks to A are long-lasting but less than permanent (are persistent)
We’ve given labor a pretty short shrift...it never varies?

Don’t we care about what happens to labor hours during recessions and booms? Isn’t that a pretty big deal?

Yes: let’s add labor.

Basic effects on labor in a one-period model:

- When income goes up, leisure and consumption both go up, labor goes down (income effect)
- When wages go up, consumption goes up, but we don’t know what happens to labor/leisure (income and substitution effects)
Variation in labor input-II

- We can amp up the substitution effect relative to the income effect on labor supply by increasing wages only for one period.

- If wages are only high today, then consumption today only goes up by a little, because the benefit is divided over many periods, so the income effect is small.

- But the substitution effect remains in full force.

- Consequently, we know that labor will go up.
Variation in labor input-II

- We can amp up the substitution effect relative to the income effect on labor supply by increasing wages only for one period.

- If wages are only high today, then consumption today only goes up by a little, because the benefit is divided over many periods, so the income effect is small.

- But the substitution effect remains in full force.

- Consequently, we know that labor will go up.

- The point: if long run (permanent wage shift) labor supply is inelastic, short run (temporary wage shift) is elastic.
Before: Demand (and supply(?)

**Figure 6.5 Clearing of the Labor Market**

- Supply curve for labor
- Demand curve for labor

$L^* = L$
Before: Demand (and supply(?))
Now: Supply and Demand

Figure 8.15: Clearing of the Labor Market
**The Good News: Model v. Reality**

- We’re trying to explain business cycles with persistent but not permanent productivity ($A_t$) shocks.
- Does it stack up with reality?

<table>
<thead>
<tr>
<th>Concept</th>
<th>Symbol</th>
<th>Reality</th>
<th>Model</th>
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<tr>
<td>Total Factor Productivity</td>
<td>$A_t$</td>
<td>↓/?</td>
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<td>Labor</td>
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<tr>
<td>Investment</td>
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<td>↓</td>
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<tr>
<td>Labor productivity</td>
<td>$\frac{L_t}{Y_t}$</td>
<td>↓</td>
<td>↓</td>
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- So far so good, with a few peculiarities & tweaks.
Differences between model and reality

- We depend on big intertemporal effects
  - But empirical studies find smaller-than-necessary effects
  - Response: shocks are persistent but not permanent
- We depend on big wage/substitution/price effects to get big cyclicality of labor
  - But some empirical studies find little response of labor to temporary wage changes
- We think labor productivity should be procyclical
  - But in reality it’s less procyclical than we’d expect.
- Takeaway: there are some quantitative puzzles, but this is going pretty well so far, everything is going in the right direction.
- Let’s add more realistic capital and unemployment/matching (Chapter 9).