Feedback via Message Passing in Interference Channels

(Invited Paper)

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***Abstract*—In distributed wireless networks, nodes often do not know the topology (network size, connectivity and the channel gains) of the network. Thus, they have to compute their transmission and reception parameters in a distributed fashion. In this paper, we consider the information required at the nodes to achieve globally optimal sum capacity. Our first result relates to the case when each of the transmitter know the channel gains of all the links that are at-most two-hop distant from it and the receiver knows the channel gains of all the links that are three-hop distant from it in a deterministic interference channel. With this limited information, we find that distributed decisions are sum-rate optimal only if each connected component is in a one-to-many configuration or a fully-connected configuration. In all other cases of network connectivity, the loss can be arbitrarily large. We then extend the result to see that O(K) hops of information are needed in general to achieve globally optimal solutions. To show this we consider a class of symmetric interference channel chain and find that in certain cases of channel gains, the knowledge of a particular user being odd user or even user is important thus needing O(K) hops of information at the nodes.**

I. Introduction

One of the fundamental challenges in mobile wireless networks is lack of complete network state information with any single node. In fact, the common case is when each node has a partial view of the network, which is different from other nodes in the network. As a result, the nodes have to make distributed decisions based on their own local view of the network. One of the key question then arises is how often do distributed decisions lead to globally optimal decisions.

The study of distributed decisions and their impact on global information-theoretic sum-rate performance was initiated in [1] for two special case of deterministic channels, and then extended to Gaussian version of those topologies in [2]. We proposed a protocol abstraction which allows one to narrow down to relevant cases of local view per node. In this abstraction, in which both transmitters and receivers participate to forward messages regarding network state information to other nodes in the network. The local message-passing allows the information to trickle through the network and the longer the protocol proceeds, the more they can learn about the network state. More precisely, the protocol proceeds in rounds, where each round consists of a message by each transmitter followed by a message in response by each receiver. Half rounds are also allowed, where only transmitters send a message. One of the main results in [2] is that with 1.5 rounds of messaging, the gap between network capacity based on distributed decisions and that based on centralized decisions can be arbitrarily large for a three-user double-Z channel (two Z-channels stacked on each other). Thus, for some channel gains, decisions based on the nodes' local view can lead to highly suboptimal network operation.

In this paper, we consider the general problem of single-hop K-user deterministic interference channels [3–6] with arbitrary network connectivity. We provide a complete characterization of all topologies which can be universally optimal with 1.5 rounds of messaging. A scheme is considered to be universally optimal if for all channel gains, the distributed decisions lead to sum-rate which is same as sum-capacity with full information. With 1.5 rounds of messaging, a transmitter knows all channel gains which are two hops away from it and a receiver knows all gains which are three hops away from it. So if the network diameter is larger than three, then no node in the network has full information about the network. Thus, while the capacity of general interference channel is still unknown (even with full network state information), we can characterize which topologies can be universally optimal with partial information.

It turns out that only those networks whose connected components are either fully-connected or have one-to-many connectivity can be universally optimal. The result is intuitively satisfying, since in both cases the nodes which need to control their transmissions to balance their own rate and interference to other receivers have *full* information about the network after 1.5 rounds. For the proof of non-existence of a universally optimal strategy, we provide the global topology information as the genie which each of the node can use to make decisions. For achievability, we give a strategy for any local topology knowledge which would be optimal when there exist a universally optimal strategy for the global topology.

We also consider a general K-user stacked Z-channel with a reduced parametrization where all direct links have identical gain and all cross-links are of the same value. Thus, there are only three unknown parameters, size of network K, direct link gain and the cross link gain. We show that one round is sufficient to achieve sum-capacity if the ratio α of cross-link gain to direct link gain is less than 1/2 (very weak interference) or greater than 2 (very strong interference). In the first case ($\alpha \leq 1/2$), flows treat interference as noise and thus learning about other parts of the network is not useful. In the second case ($\alpha \geq 2$), interference can be completely cancelled out and thus each node can be greedy without requiring any information from other nodes. For $\alpha \in (1/2, 2/3]$, no more than two rounds are required to achieve optimality. And for all other values of $\alpha \in (2/3, 2)$, $O(K)$ rounds are required to achieve optimality. This suggests that network could measure itself adaptively by using more rounds only if certain channel gains are detected in the first round.

The rest of the paper is organized as follows. In Section II, we formulate the problem and provide the message passing protocol and some definitions in Section III. In Section IV, we find the connectivities for which a universally optimal strategy exists with 1.5 rounds of message passing protocol. Section V extends the results to more rounds of message passing in a K user symmetric stacked Z-channel.

II. Problem Formulation

Consider a deterministic interference channel with K transmitters and K receivers, where the inputs at k^{th} transmitter in time i can be written as $X_k[i] = \left[X_{k_1}[i] \ X_{k_2}[i] \ \ldots X_{k_q}[i]\right]^T$, $k = 1, 2, \cdots, K$, such that $X_{k_1}[i]$ and $X_{k_q}[i]$ are the most and the least significant bits, respectively. The received signal of user j, $j = 1, 2, \cdots, K$, at time i is denoted by the vector $Y_j[i] = \left[Y_{j_1}[i] \ Y_{j_2}[i] \ \ldots \ Y_{j_q}[i]\right]^T$. Associated with each transmitter k and receiver j is a non-negative integer n_{kj} that defines the number of bit levels of $\mathbf{X}_k$ observed at receiver j. The maximum level supported by any link is $q = \max_{j,k}(n_{jk})$. The network can be represented by a square matrix H whose $(j,k)^{th}$ entry is n_{jk}. Note H need not be symmetric.

Specifically, the received signal $Y_j[i]$ is given by

$$Y_j[i] \quad = \quad \sum_{k=1}^{K} \mathbf{S}^{q-n_{kj}} X_k[i] \qquad (1)$$

where $\oplus$ denotes the XOR operation, and $\mathbf{S}^{q-n_{jk}}$ is a $q \times q$ shift matrix with entries $\mathbf{S}_{m,n}$ that are non-zero only for $(m, n) = (q - n_{jk} + n, n), n = 1, 2, \ldots, n_{jk}$.

We now define network state and network connectivity. We assume that that there is a direct link between every transmitter T_i and its intended receiver D_i. On the other hand, if a cross-link between transmitter i and receiver j does not exist, then $H_{ij} \equiv 0$. Given a network, its connectivity is a set of edges $\mathsf{E} = \{(\mathsf{T}_i, \mathsf{D}_j)\}$ such that a link $\mathsf{T}_i - \mathsf{D}_j$ is not identically zero. Then the set of network states, $\mathcal{G}$, is the set of all weighted graphs defined on E. The set of network states can be written as

$$\mathcal{G}(\mathsf{E}) = \{H : H_{ij} \equiv 0 \text{ if } (\mathsf{T}_i, \mathsf{D}_j) \notin \mathsf{E} \text{ else } H_{ij} \in \{0, 1, \ldots, q\}\}$$

Note that the channel gain can be zero but not guaranteed[1] to be if the node pair $(\mathsf{T}_i, \mathsf{D}_j) \in \mathsf{E}$.

We assume that none of the channel coefficients in the matrix H, or even the size of matrix H is known before the start of the message passing protocol. As a result, none of the nodes are aware of the maximum possible transmission rates and the associated coding schemes to achieve the capacity. The decision taken by the nodes only depend on the information that the nodes possess. Our objective is to understand the impact of nodes' decisions on network sum-rate, when the decisions are based on their partial information about the matrix H. For transmitters we will denote this partial information about the network as N_k and as N'_k for the receivers. If the nodes know nothing about the network matrix H (i.e no information about its size or entries), then $N_k = N'_k = \Phi$ (empty set), which is equal to assuming that there is no other node in the network. On the other hand, if the nodes know everything about the network, then $N_k = N'_k = H$ and is also the most commonly assumed scenario in most information-theoretic analyses [5, 7].

In Section III, we will define a special trajectory of sequence of growing network information which is directly connected to protocols in practical systems and is also related to commonly used metric of 'number of hops' to denote amount of side information at each node. To aid analysis, we will assume that all nodes are provided some side information, SI, about the network state before the onset of the protocol. Thus, nodes may have non-zero information about the network before even a single message is sent.

III. Message Passing Protocol

For nodes to learn and propagate the network state, they have to communicate with each other. This inter-node communication is possible only with nodes to which there is a direct link, i.e, messages have to be exchanged locally and those messages are then processed and propagated to other nodes. This obvious construct of local message passing is central to all multi-hop network protocols. In our development, the only practical reality we will be concerned with is that direct communication is possible only between neighbors and its impact on amount of network state information at each node. Hence, we will simplify some of the implementation complexities as follows.

The proposed message passing protocol proceeds in rounds, where each full round has two phases: a *forward* phase where all transmitters broadcast a message and a *reverse* phase where all receivers broadcast a message each. We assume that all messages are scheduled so that there are no "collisions" at

[1]The model is inspired by fading channels, where the existence of a link is based on its average channel gain. On the average the link gain may be above noise floor but its instantaneous value can be below noise floor.

any of the nodes in receiving mode due to simultaneous transmissions. Finally, the broadcast messages can only be heard by nodes to which the sending nodes has direct links (the links that are in the network connectivity E), thus no extra feedback or Genie channels are available.

The message broadcasted by the transmitter k in round t (transmitters are data sources) is labeled $m_{k,t}$, which is received by all the receivers j who have direct links to transmitter k. Analogously, the message broadcasted by the receiver k at round t is labeled $M_{k,t}$, which is received by all the transmitters j who have a direct link to receiver k.

The message passing protocol is described below [1, 2].

1) **Round 1 (Forward)**: The first message from each transmitter is a known training signal along with the transmitter identity. Thus $m_{k,1} = \{\psi_k, \mathsf{T}_k\}$, where ψ_k is the training signal from transmitter k. At the end of the transmitter messages, receiver j knows the channel gains from all the transmitters to which it is connected.
Round 1 (Reverse): The receiver k broadcasts $M_{k,1} = \bigcup_{i\in \mathsf{E}_k} \{(H_{i,k}, \mathsf{T}_i, \mathsf{D}_k)\}$, where E_k is the set of vertices connected to receiver k. Transmitter T_j can receive $M_{k,1}$ if it has a direct link to receiver k. This completes the first round.
2) **Round** $t > 1$: In round $t > 1$, nodes only forward new information which is computed as follows. In the forward phase for transmitters, the broadcast message is
$$m_{k,t} = \bigcup_{j\in \mathsf{J}_k} M_{j,t-1} \setminus \bigcup_{t'=2}^{t-1} m_{k,t'} \setminus \bigcap_{j\in \mathsf{J}_k} \left\{ \bigcup_{t'=1}^{t-1} M_{j,t'} \right\}, \tag{2}$$
where J_k is the set of vertices connected to transmitter k. The message $m_{k,t}$ is a concatenated version of its received messages from previous round minus the messages it has broadcasted in previous transmissions and those that are already known to all of its neighbors. In response, the receivers broadcasts following in the reverse phase
$$M_{k,t} = \bigcup_{j\in \mathsf{E}_k} m_{j,t} \setminus \bigcup_{t'=1}^{t-1} M_{k,t'} \setminus \bigcap_{j\in \mathsf{E}_k} \left\{ \bigcup_{t'=2}^{t} m_{j,t'} \right\}. \tag{3}$$
The message $M_{k,t}$ is the concatenation of its received message minus its previously broadcasts messages and after removing what is known to all its neighboring transmitters. The messages $m_{k,t}$ and $M_{k,t}$ are similar to the extrinsic information in belief propagation with the main difference being that the messages are broadcasts.
3) **Stopping Rule**: If a transmitter or receiver has no new updates, it sends a NULL message ϕ in its assigned time-slot. Thus, nodes only forward information when new information is received and send "nothing" otherwise. When all the neighbors of a node send a NULL message, each node stops sending any new messages (even NULL messages).

We now formally define the concept of universally optimal strategy with partial information. Suppose that the transmitter i knows local information N_i, the receiver i knows local information N_i' and all the nodes know side information SI.

Definition 1 (Universal Optimality [2])**.** *A universally optimal strategy with partial information at nodes (N_i at transmitter i, N_i' at receiver i and SI at all the nodes) is defined as the strategy that each of the transmitter i uses based on its local information N_i and side information SI, such that following holds. The strategy yields a sequence of codes having rates R_i at the transmitter i such that the error probabilities at the receiver, $\lambda_1(n), \cdots \lambda_K(n)$, go to zero as n goes to infinity, satisfying*
$$\sum_i R_i = C_{sum}$$
for all the sets of network states consistent with the side information. Here C_{sum} is the sum-capacity of the whole network with the full information.

IV. Existence of Universally Optimal Strategies with 1.5 rounds of Message Passing Protocol

In this section, we find the condition on network connectivities for which there exist a universally optimal strategy with 1.5 rounds of message passing. To give the condition, we first define the two sets of configuration of network connectivities called one-to-many configuration or a fully-connected configuration.

Definition 2 ([8])**.** *A network connectivity of K users is in one-to-many configuration if there are $2K-1$ links in the network connectivities that include one of the transmitters connected to all the receivers while all other transmitters only connected to their own receivers.*

Definition 3 ([8])**.** *A network connectivity of K users is in fully-connected configuration if there are K^2 links in the network connectivity with each transmitter connected to all the receivers.*

The next theorem describes our main result of this Section. Suppose that each node only knows the local network connectivity information, local channel gains and the local node identities. We find the topologies for which a universally optimal strategy exist. The outer bound provides a genie aided topology information to all the nodes while the achievable strategy assumes only the local information.

Theorem 1 ([8])**.** *Suppose that each node knows only the network connectivity information (E) provided by 1.5 rounds of message passing protocol. Then, there exist a universally optimal strategy for a K-user interference channel with 1.5 rounds of message passing if and only if all the connected components of the network connectivity are in one-to-many configuration or fully-connected configuration.*

Proof: We find that the topologies in which there is a connected component that is not in one-to-many configuration or in fully-connected information, universally optimal strategy does not exist even with the genie-aided knowledge of global network connectivity. For the achievability, the following strategy for the various local network connectivity information seen by a user achieves the optimal sum capacity if the network connectivity is as in the statement of the theorem and is an achievable strategy otherwise.

1) One-to-many network connectivity with L nodes and the current node has degree 1: The transmitter sends at a rate of n_{ii}.
2) One-to-many network connectivity with L nodes and the current node has degree L: The transmitter sends at the signal levels that do not potentially create interference to all the users that it interferes.
3) Fully connected network connectivity with L nodes: The node uses the node identities to get its ordering in L nodes and uses a pre-decided strategy that will be optimal for fully connected L node topology.
4) Any other local information: The node sends at a rate 0, or in other words remain silent.

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Consider the set of all possible $2^{K(K-1)}$ network connectivities for a K-user interference channel with the direct links from each transmitter to its receiver always connected. Out of these, let D_{opt} be the number of possible connectivities for which there exist a universally optimal strategy for a K-user interference channel with 1.5 rounds of message passing. The next theorem gives approximate characterization of D_{opt}.

Theorem 2. *The value of D_{opt} can be bounded from above and below by*

$$B_K \leq D_{opt} \leq B_K(K+1)^K, \tag{4}$$

where B_K is the K^{th} Bell number [9].

Proof: To prove $D_{opt} \geq B_K$, we just count the number of connectivities in which all components are in fully connected configuration. To prove the outer bound of $D_{opt} \leq B_K(K+1)^K$, we consider a split of the K user connectivity into k connected subcomponents. There are $S(K,k)$ number of subcomponents, where $S(K,k)$ is the Stirling number of the second kind. In each of the possible split into k subcomponent, each subcomponent can have (number of nodes in component + 1)$\leq K+1$ possible choices of a good topology. Thus,

$$\begin{aligned} D_{opt} &\leq \sum_{k=1}^{K} S(K,k)(K+1)^k \\ &\leq \sum_{k=1}^{K} S(K,k)(K+1)^K = B_K(K+1)^K \end{aligned}$$

■

Note that $\frac{\log(B_K)}{K\log K} \to 1$ as $K \to \infty$ [10]. Thus, $1 \leq \lim_{K\to\infty} \frac{\log(D_{opt})}{K\log K} \leq 2$. This shows that the optimal connectivities scale as $2^{K\log(K)}$ while the total are of the order of 2^{K^2} thus proving that the number of optimal connectivities to the total connectivities goes to zero.

V. Universally Optimal Strategies for a Symmetric Interference Channel Chain

While exact analysis for two and three-user interference channels was tractable, extensions to general interference channel remains out of reach at the current moment. To make progress, we will consider a special case of channel gains which reduces the network parametrization to only three unknowns: the number of nodes in the network, and two channel gain parameters representing the direct link and the cross link. In addition, we will assume that the network connectivity is of the form of $K-1$ Z-channels stacked on top of each other as described below. Our objective is to quantify the achievable sum-rates with limited number of rounds of message passing.

A. Channel Models and Message Passing Protocol

For the deterministic model characterized by the direct channel gain n and the cross channel gain m, the received signal Y_{ji}, $j = 1, 2, \cdots, K$ of K-user Z-channel is given by

$$\begin{aligned} Y_{1i} &= \mathbf{S}^{q-n} X_{1i} \\ Y_{ji} &= \mathbf{S}^{q-m} X_{(j-1)i} \oplus \mathbf{S}^{q-n} X_{ji} \text{ for all } 2 \leq j \leq K. \end{aligned}$$

We label the top transmitter as Node 1, the next transmitter as Node 2 and so on. Thus, we would use the phrase "node above" and "node below" with respect to this labeling unless otherwise stated. We assume that all users knows that it is a symmetric Z-channel but do not know n, m and their relative position in the network (which also includes the information about K). In this case, the message passing protocol can be performed with less message content than sending the whole channel matrix as explained in [2]. We will now provide achievability with d rounds of message passing algorithm for $d \leq K$. Since the users do not know K and their placement in the network, the decisions have to be made in a distributed fashion.

We assume that the nodes have the network connectivity and its reduced parametrization as side information, i.e, $\mathsf{SI} = \{\mathcal{G}', H_{ii} = H_{11}, H_{i,i+1} = H_{12}, \alpha = H_{12}/H_{11}\}$, where $\mathsf{F} = \{\mathsf{E} : (\mathsf{T}_i, \mathsf{D}_j)\}$ and $\mathcal{G}' = \{H : H_{ij} \equiv 0 \text{ if } \text{ if } j \neq i \text{ or } j \neq i+1, H_{11} = H_{22} = \cdots \in \{0, 1, \ldots, q\}, H_{12} = H_{23} = \cdots \in \{0, 1, \ldots, q\}\}$. However, they do not know the network size K or the direct channel gains $\{H_{11}\}$.

B. Results

Our first result shows that one round of message passing achieves the sum capacity for $\alpha \geq 2$ or $\alpha \leq 1/2$.

Theorem 3 ([2])**.** *For $\alpha \geq 2$ and $\alpha \leq 1/2$, there exists a universally optimal strategy with one round of message passing protocol for any K if each node knows the side information that the network state is symmetric and is parameterized by three parameters K, n and $\alpha = m/n$, and the value of α.*

In Theorem 3, we proved that for $\alpha \geq 2$ or $\alpha \leq 1/2$, one round was sufficient for the existence of a universally optimal strategy. Further, one round is the minimum needed for the transmitters to know the rate at which they need to transmit. These regimes cover the very strong and the very weak interference. Note that the result of interference to be treated as noise is optimal for the point of sum-capacity only holds when $\alpha \leq 1/2$for $K \geq 3$ [2] which restricts the choice to $\alpha \leq 1/2$ in the above theorem.

We next consider the case when the interference is strong but not very strong, and when the interference is weak but not very weak. In this case, we give a strategy that uses O(K) rounds to achieve the sum capacity. This achievable strategy uses O(K) rounds because the nodes base their decisions on their placement in the network. More precisely, the nodes learn if they are the even-numbered nodes or the odd-numbered nodes in the network state to decide on the rate allocation.

Theorem 4 ([2]).

1) *For $1/2 < \alpha \leq 2/3$, there exists a universally optimal strategy with 2 rounds of message passing protocol for any K.*
2) *For $2/3 < \alpha < 1$, the sum-rate of $n + (K-1)(n-m)$ can be achieved with 1 round of message passing, which is optimal for $K < 3$. In general, the sum-rate of $n + (K-1)(n-m) + (2m-n)\sum_{i=1}^{\lfloor (d-1)/2 \rfloor}(1_{K \geq 2i+1}) + (n-m)\sum_{i=1}^{\lfloor (d-2)/2 \rfloor}(1_{K \geq 2i+2})$ can be achieved in $d.5$ rounds. Thus, this strategy achieves optimal sum-rate for $K \geq 3$ in $K.5$ rounds.*
3) *Let $1 \leq \alpha < 2$, then the sum-rate of $n + (K-1)(m-n) + (2n-m)\sum_{i=1}^{\lfloor (d-1)/2 \rfloor}(1_{K \geq 2i+1})$ can be achieved in $d \geq 1$ rounds. This strategy is optimal with one round of message passing for $K < 3$. Further, this strategy is optimal for any odd $K \geq 3$ with K rounds, and for any even K, $K \geq 4$ with $(K-1)$ rounds.*

For $1/2 < \alpha \leq 2/3$, 2 rounds of message passing are enough since it is enough for the transmitters to learn if there are more than two nodes above it. However for $2/3 < \alpha < 2$, $O(K)$ rounds were needed to converge to the sum capacity in general so that the transmitters are able to know their placement in the channel connectivity. More precisely, the nodes learnt their relative position in the network being even/odd user. This requires $O(K)$ rounds of message passing. We note that this scheme involved the use of public as well as a private message at the transmitters.

O(K) rounds of message passing are needed for the existence of universally optimal strategy in the strong but not very strong interference because the consideration to even/odd number of users is important. Suppose that the nodes do not use node identity in the choice of strategies, but only the relative position known through the local information. The intuition in the use of O(K) rounds of message passing is that if constant number of neighboring nodes (let us say 10, numbered $2K_1+1, \cdots, 2K_1+10$) know the exact similar structure (which would happen with less than O(K) rounds), they choose the same rate. If this constant rate is not $m/2$, the sum rate will not be optimal. Further, pick out even number of users (say $2K_1$) from above and the sum rate of these top nodes has to be $K_1 m$, otherwise it won't lead to global optimality. Similarly, choose $2K_2$ users from below such that $K - 2K_2 = 2K_1 + 9$ or $2K_1 + 10$ depending on K being odd/even. The sum rate for these chosen nodes from below has to be $K_2 m$ for optimality. Thus, if remaining nodes $(K - 2(K_1+K_2))$ are odd, the sum rate is not optimal. Thus, there cannot exist an optimal rate allocation with less than O(K) rounds of message passing.

VI. Conclusions

We consider a general deterministic interference channel in which the nodes know the channel gains through a message passing protocol. With 1.5 rounds of this protocol, each transmitter learns the channel gains of all the links that are at-most two-hop distant from it and the receivers learn the channel gains of all the links distant at-most three-hops from it. With this limited information, this paper classifies all interference channel topologies based on their ability to support distributed strategies which are universally optimal. We also note that the genie-aided information regarding global network connectivity do not aid in making a topology able to support a universally optimal strategy. These results have been extended to consider cooperation between transmitters or receivers in [11].

Seeking universal optimality, where local decisions with certain side information are always globally optimal, we discovered that there appears to be a critical minimum information required for the network to allow globally optimal decisions. We considered a class of stacked Z-channel to see that in certain cases, 1 or 2 rounds were sufficient while in certain others O(K) rounds were required to achieve globally optimal solution.

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