Normalized Sum-Capacity of Interference Networks with Partial Information

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***Abstract*—In distributed wireless networks, nodes often do not have access to complete network information (*e.g.* network topology, channel gains, etc.). As a result, they have to execute their transmission and reception strategies with partial information about the network, in a distributed fashion. Thus, the key question is how good are the distributed decisions in comparison to the optimal decisions based on full network knowledge.**

In this paper, we formalize the concept of partial-information sum-capacity by defining *normalized sum-capacity*, which is defined as the maximum achievable fraction of full-information sum-capacity with a given amount of partial information. We then examine four deterministic networks, multiple access, multiuser Z-channel chain, one-to-many and many-to-one interference channel, and characterize the normalized sum-capacity. For each network, two cases of partial network information are analyzed: (a) each transmitter only knows the channel gains to its receiver, and (b) transmitters knows the channel gains of all links which are no more than two hops away. Quite interestingly, we show that in all eight cases (4 networks $\times$ 2 forms of partial information), the normalized sum-capacity is achieved by scheduling subnetworks for which there exist a universally optimal distributed strategy with the available partial information. Furthermore, we show that while actual sum-capacity is not known in all cases, normalized sum-capacity can be in fact be exactly characterized.

I. Introduction

One of the fundamental challenges in mobile wireless networks is lack of complete network state information at any single node. In fact, the common case is when each node has a partial view of the network, which is different from other nodes in the network. As a result, the nodes have to make distributed decisions based on their own local view of the network. One of the key question is *how good are distributed decisions compared to (essentially centralized) case of network operation with full channel information about all the links?*

To answer the above question, we define normalized sum-capacity which is the maximum fraction of full-information sum-capacity that can be achieved with the given amount of partial information. The normalized sum-capacity is upper bounded by one, since no partial information scheme can do better than the case of full information. With partial information, the set of codes which can be used by each transmitter and the decoder which can be used by their corresponding receivers are limited by their respective partial information. Thus, no node can base its actions on what it does *not* know.

To achieve normalized sum-capacity of one, the distributed decisions based on partial information have to be optimal (*i.e.* achieve the sum-capacity of full-information case) for all values of unknown channel gains. If a distributed protocol achieves normalized sum-capacity of one, then such schemes are labeled as *universally optimal* [1]. In [1, 2], we showed that for some cases of partial information, normalized sum-capacity is strictly less than one for many networks. However, normalized sum-capacity was not computed for any network in [1–3].

In this paper, we compute the normalized sum-capacity of four deterministic networks [4] with two different forms of partial information. Specifically, the four networks are the multiple access channel, K-user Z-channel chain (Z-channels stacked on top of each other), one-to-many and many-to-one interference channels. Two forms of partial information are studied. In the first case, each transmitter only knows the direct channel gain to its intended receiver. This is inspired by the SINR model [5], where only channel gain for the direct link is known and interference is treated as noise. The second case is inspired by our prior work in [1, 3], where we proposed a protocol abstraction based on local message passing. In the message passing terminology [3], we study the case of 1.5 rounds of message passing, which results in transmitter knowledge of all channels within two hops and receiver knowledge of all channel gains which are three-hops away. The case of 1.5 message rounds was analyzed in [2], where we showed that universal optimality (normalized sum-capacity of one) can be achieved by only those K-user bipartite networks if every connected component is either fully-connected or one-to-many network.

For the first case of partial information, we show that spatial-reuse maximizing non-interfering link scheduling achieves the highest possible normalized sum-capacity. Thus, maximum weight independent set (MWIS) scheduling [5, 6] is information-theoretically optimal if the transmitters only know the direct channel gains to their respective receivers. For all four networks, normalized sum-capacity is less than one unless the network has only one user (exact normalized sum-capacity is computed in each case). This is the first information-theoretic result that proves optimality of scheduling in these channels from a sum-capacity viewpoint.

For the case of 1.5 rounds of message passing, we show that the maximum weight independent set (MWIS) scheduling is strictly sub-optimal. We propose a new *sub-network scheduling* algorithm which is based on scheduling over all sub-networks which can be universally optimal with 1.5 rounds of network information (as characterized in [2]). Furthermore, we show that the sub-network scheduling algorithm achieves the normalized sum-capacity in each of the four networks (with 1.5 rounds of network information). We prove this by deriving an upper-bound on the normalized sum-capacity and showing that in each case there exists a schedule which achieves the upper-bound. Hence, we also characterize the normalized sum-capacity in each network (with 1.5 rounds of message passing). This is quite interesting, as we characterize the normalized sum-capacity without requiring to know the sum-capacity with full information. In fact, the sum-capacity (with full-information) is still unknown for the general K-user Z-channel chain.

Our results also allow us to gauge the utility of more network information on achievable sum-capacity. For instance, we show that in the three-user Z-channel chain with two-hops of network knowledge, sub-network scheduling achieves normalized sum-capacity of at-least $2/3$ while the conventional MWIS enforces flows to be non-interfering and thus cannot achieve more than $1/2$.

The rest of the paper is organized as follows. In Section II, we give some preliminaries and definitions that will be used throughout the paper. In Section III, IV, V and VI, we present our main results on MAC channel, Z-channel chain, one-to-many interference channel and many-to-one interference channel respectively, we discuss the scheduling based achievability and conclude the paper in Section VII.

II. MODEL, CODES AND CAPACITY

A. Channel Model

We consider the deterministic [4] interference channel with K transmitters and K receivers, where the input of the k^{th} transmitter at time i can be written as $X_k[i] = \left[X_{k_1}[i]\ X_{k_2}[i]\ \ldots X_{k_q}[i]\right]^T$, $k = 1, 2, \cdots, K$, such that $X_{k_1}[i]$ and $X_{k_q}[i]$ are the most and the least significant bits, respectively. The received signal of user j, $j = 1, 2, \cdots, K$, at time i is denoted by the vector $Y_j[i] = \left[Y_{j_1}[i]\ Y_{j_2}[i]\ \ldots\ Y_{j_q}[i]\right]^T$. Associated with each transmitter k and receiver j is a non-negative integer n_{kj} that represents the gain of the channel between them. The received signal $Y_j[i]$ is given by

$$Y_j[i] \quad = \quad \sum_{k=1}^{K} \mathbf{S}^{q-n_{kj}} X_k[i] \qquad (1)$$

where q is the maximum of the channel gains (*i.e.* $q = \max_{j,k}(n_{jk})$), the summation is in $\mathbb{F}_2^q$, and $\mathbf{S}^{q-n_{jk}}$ is a $q \times q$ shift matrix with entries $\mathbf{S}_{m,n}$ that are non-zero only for $(m, n) = (q - n_{jk} + n, n), n = 1, 2, \ldots, n_{jk}$.

The network can be represented by a square matrix H whose $(j,k)^{th}$ entry is n_{jk}. Note H need not be symmetric. We assume that that there is a direct link between every transmitter T_i and its intended receiver D_i. On the other hand, if a cross-link between transmitter i and receiver j does not exist, then $H_{ij} \equiv 0$. Given a network, its network is a set of edges $\mathsf{E} = \{(\mathsf{T}_i, \mathsf{D}_j)\}$ such that a link $\mathsf{T}_i - \mathsf{D}_j$ is not identically zero. Then the set of network states, $\mathcal{G}$, is the set of all weighted graphs defined on E. Note that the channel gain can be zero but not guaranteed[1] to be if the node pair $(\mathsf{T}_i, \mathsf{D}_j) \in \mathsf{E}$.

We will consider four networks: Multiple-Access, Z-channel chain, one-to-many and many-to-one networks. In Z-channel chain, $(\mathsf{T}_i, \mathsf{D}_j) \notin \mathsf{E}$ for all $j \neq i$ or $i + 1$. In one-to-many network, $(\mathsf{T}_i, \mathsf{D}_j) \notin \mathsf{E}$ for all $i \neq 1$ and $j \neq i$. In many-to-one network, $(\mathsf{T}_i, \mathsf{D}_j) \notin \mathsf{E}$ for all $j \neq K$ and $j \neq i$.

B. Normalized sum-capacity

For each user k, let message index m_k be uniformly distributed over $\{1, 2, ..., 2^{nR_k}\}$. The message is encoded as X_k^n using the encoding functions $e_k(m_k|N_k, \mathsf{SI}) : \{1, 2, \ldots, 2^{nR_k}\} \mapsto \{0, 1\}^{nq}$, which depend on the local view, N_k, and side information about the network, SI. The message is decoded at the receiver using the decoding function $d_k(Y_k^n|N'_k, \mathsf{SI}) : \{0, 1\}^{nq} \mapsto \{1, 2, \ldots, 2^{nR_k}\}$, where N'_k is the receiver local view and SI is the side information. The corresponding probability of decoding error $\lambda_k(n)$ is defined as $\Pr[m_k \neq d_k(Y_k^n|N'_k, \mathsf{SI})]$. A rate tuple $(R_1, R_2, \cdots, R_K)$ is said to be achievable if there exists a sequence of codes such that the error probabilities $\lambda_1(n), \cdots \lambda_K(n)$ go to zero as n goes to infinity for all network states consistent with the side information. The sum-capacity is the supremum of $\sum_i R_i$ over all possible encoding and decoding functions.

We will now define normalized sum-rate and normalized sum-capacity that will be used throughout the paper. These notions represent the percentage of the full-knowledge sum-capacity that can be achieved with partial-information.

Definition 1. *Normalized sum-rate of α is said to be achievable in a set of network states with partial information if there exists a strategy that each of the transmitter i uses based on its local information N_i and side information SI, such that following holds. The strategy yields a sequence of codes having rates R_i at the transmitter i such that the error probabilities at the receiver, $\lambda_1(n), \cdots \lambda_K(n)$, go to zero as n goes to infinity, satisfying*

$$\sum_i R_i \geq \alpha C_{sum}$$

for all the sets of network states consistent with the side information. Here C_{sum} is the sum-capacity of the whole network with the full information.

[1]The model is inspired by fading channels, where the existence of a link is based on its average channel gain. On the average the link gain may be above noise floor but its instantaneous value can be below noise floor.

Definition 2. *Normalized sum-capacity, α^*, is defined as the supremum over all achievable normalized sum rates α.*

Earlier, we defined a notion of *universal optimality* [1], which is now a special case of our current definition. A universally optimal strategy is simply the one which achieves $\alpha^* = 1$ in the desired network. Hence, our main result in [2] characterizes *all* single-hop interference networks which has normalized sum-capacity equal to one ($\alpha^* = 1$) with 1.5 rounds of message passing (two-hop channel knowledge at transmitter and three-hop channel knowledge at receivers):

Theorem 1 ([2]). *The normalized sum-capacity of a K-user interference channel, when each transmitter knows the link gains of all the links that are at-most two-hop distant from it, is equal to one (*i.e. $\alpha^* = 1$*) if and only if all the connected components of the topology are in one-to-many configuration or a fully-connected configuration.*

This Theorem basically gives us all networks in which there is no loss in capacity due to the considered partial-information. However, it does not specify the loss (or equivalently the normalized sum-capacity) in all other cases. In this paper, we take a step further and exactly characterize the normalized sum-capacity in the aforementioned four networks, in two cases: (a) transmitters have only direct link gain information to its intended receiver, and (b) transmitters have knowledge of all channels within two-hops away. In both the cases, the receiver information would be assumed as the union of the information of all the transmitters to which it is connected. The side information (SI) of network connectivity is always assumed at all nodes.

III. Network 1: Multiple Access Channel

We start with a simple example to illustrate the concepts that we defined in Section II. We consider the K-user multiple access channel when each transmitter only knows the gain of the channel from itself to the receiver (User i has link gain n_i.). In this case, we show that the normalized sum-capacity is $1/K$ which can be achieved by simply scheduling one user at a time in a total of K time-slots.

The main challenge is to show the converse. Let $K > 1$, as otherwise the result holds trivially. Assume that normalized sum-rate of $\alpha > 1/K$ is achievable. Then, we should be able to achieve a rate tuple satisfying

$$R_i > \frac{1}{K} n_i, \ \forall \ 1 \le i \le K. \tag{2}$$

This is because each node is unaware of the other channel gains. To achieve a normalized sum-rate larger than $\frac{1}{K}$, each user should send at a rate larger than a fraction $\frac{1}{K}$ of its channel gain (otherwise in the case when all other channel gains are zero, achievable normalized sum-rate is smaller than $\frac{1}{K}$). Now, we will show that this rate-tuple cannot be achieved. With the capacity bound of full information,

$$\begin{aligned} R_K &\le \max_{1 \le i \le K} n_i - \sum_{i=1}^{K-1} R_i \\ &\overset{(2)}{<} \max_{1 \le i \le K} n_i - \frac{1}{K} \sum_{i=1}^{K-1} n_i. \end{aligned} \tag{3}$$

Since the K^{th} transmitter does not know n_i for $1 \le i \le K-1$,

$$\begin{aligned} R_K &< \min_{n_i, 1 \le i \le K-1} \left[\max_{1 \le i \le K} n_i - \frac{1}{K} \sum_{i=1}^{K-1} n_i \right] \\ &\le n_K - \frac{K-1}{K} n_K = \frac{1}{K} n_K \end{aligned} \tag{4}$$

Thus, $R_K < \frac{n_K}{K}$, which contradicts (2) and hence $\alpha^* \le \frac{1}{K}$.

Since all the links are at-most two hops from each transmitter, the normalized sum-capacity in the case when each transmitter knows all the links that are at-most two hop distant from it is 1.

IV. Network 2: Z-channel chain

In this section, we consider the Z-channel chain.

A. Direct link knowledge

Theorem 2. *The normalized sum-capacity of a Z-channel chain when each of the transmitter knows only its direct link to the receiver is,*

$$\alpha^* = \begin{cases} 1 & \text{if } K = 1, \\ 1/2 & \text{if } K > 1. \end{cases} \tag{5}$$

Proof: When $K = 1$, the interference channel is a point-to-point channel with global information and the same sum-capacity as with the global information can be achieved.

We will first prove the converse for $K = 2$. Assume that a normalized sum rate of $\alpha > 1/2$ is achievable, then

$$R_i > \frac{1}{2} n_i, \ \forall \ 1 \le i \le 2. \tag{6}$$

Now, we will show that this rate-tuple cannot be achieved. The compound sum-capacity of the two user Z-channel when none of the nodes know n_{12} is an outer bound. Thus,

$$\begin{aligned} R_1 + R_2 &\le \min_{n_{12}} \max(n_{11}, n_{22}, n_{12}, n_{11} + n_{22} - n_{12}) \\ &= \max(n_{11}, n_{22}) \end{aligned} \tag{7}$$

This can be rewritten as

$$\begin{aligned} R_1 &\le \max(n_{11}, n_{22}) - R_2 \\ &< \max(n_{11}, n_{22}) - n_{22}/2 \end{aligned} \tag{8}$$

Since the first transmitter T_1 does not know the value of n_{22},

$$\begin{aligned} R_1 &< \min_{n_{22}} [\max(n_{11}, n_{22}) - n_{22}/2] \\ &\le n_{11} - n_{11}/2 = \frac{1}{2} n_{11} \end{aligned} \tag{9}$$

Thus, $R_1 < n_{11}/2$ which contradicts the assumption. Hence $\alpha^* \le 1/2$.

Now, the same upperbound ($\alpha^* \leq 1/2$) also holds for the case of $K > 2$. This is because we can consider the case that the rest of the $K-2$ users have 0 channel gains to their receivers and this is known globally known at the transmitters. This is simply the 2-user Z-channel which we showed that $\alpha^* \leq 1/2$. Hence the normalized sum-rate capacity is upper bounded by $1/2$ for all $K > 1$.

For the achievability, consider the data transfer over two time slots. In the first time-slot, even users transmit data while in the second time-slot, odd users transmit data. This way the sum rate of $\sum_{i=1}^{K} n_{ii}/2$ is achieved which is at-least half the sum-capacity with global information and thus normalized sum-capacity of $1/2$ can be achieved. ■

B. Two hop link knowledge

Theorem 3. *The normalized sum-capacity of a Z-channel chain when each of the transmitter knows all the links that are within 2-hop distant from it is,*

$$\alpha^* = \begin{cases} 1 & \text{if } K \leq 2, \\ 2/3 & \text{if } K > 2. \end{cases} \tag{10}$$

Proof: By Theorem 1 the normalized sum-capacity is 1 for $K \leq 2$. So, we will only consider $K \geq 3$ in the proof.

For the converse, we take $K = 3$ and all the rest of the direct channel gains be 0 and known to all. If all the channel gains are all n, for $\alpha > 2/3$, $R_1 < n_1/3$ since it can happen from its point of view that $n_{33} = 0$ in which case it has to assume that the second transmitter is sending at rate $> 2/3n$. Thus, the sum-rate is $< 4/3n = 2/3(2n)$ which contradicts $\alpha > 2/3$.

For the achievability, consider the data transfer over three time slots as shown in Figure 1. In the i^{th} time slot, all users $3j + i$ remain silent for all integers i, j satisfying $1 \leq j < \infty$ and $1 \leq i \leq 3$. In each time-slot, the effective topology has all its connected subcomponents in the one-to-many configuration or in the fully connected configuration and each user uses the universally optimal strategy in this case. Let $(R_1, R_2, \cdots, R_K)$ be any point in the global information capacity region. In the i^{th} time-slot, sum-rate of at least $\sum_{1\leq j\leq K, j\neq i \bmod 3} R_j$ can be achieved. Hence, the effective sum rate of at least $2/3 \sum_{1\leq i\leq K} R_i$ (or the normalized sum-rate of at least $\frac{2}{3}$) can be achieved. ■

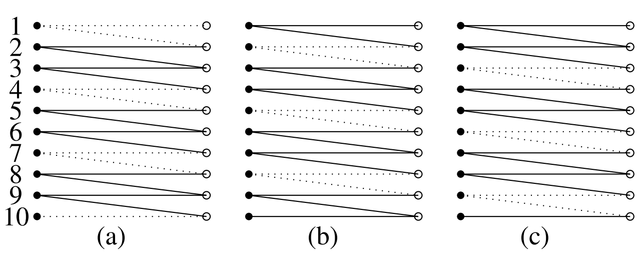

Fig. 1. Scheduling of nodes in Z-channel chain in three time-slots with two hop knowledge at transmitters, where the dotted lines represent that the transmitter is not sending data.

V. Network 3: One-to-many interference channel

In this section, we consider the one-to-many interference channel.

A. Direct link knowledge

Theorem 4. *The normalized sum-capacity of a one-to-many interference channel when each of the transmitter knows only its direct link to the receiver is,*

$$\alpha^* = \begin{cases} 1 & \text{if } K = 1, \\ 1/2 & \text{if } K > 1. \end{cases} \tag{11}$$

Proof: When $K = 1$, the interference channel is a point-to-point channel with global information and the same sum-capacity as with the global information can be achieved.

The converse for $K > 1$ follows on the same lines as the proof of Theorem 2 since for $K = 2$, we get a Z-channel in which case the Theorem 2 holds and the same arguments as in Theorem 2 for $K > 2$ apply.

For the achievability, consider the data transfer over two time slots. In the first time-slot, only the first user transmits while in the second time-slot, all but the first user transmits data. This way the sum rate of $\sum_{i=1}^{K} n_{ii}/2$ is achieved which is at-least half the sum-capacity with global information and thus normalized sum-capacity of $1/2$ can be achieved. ■

B. Two hop link knowledge

Theorem 5. *The normalized sum-capacity of a one-to-many interference channel when each of the transmitter knows all the links that are within 2-hop distant from it is,*

$$\alpha^* = 1 \text{ for all } K > 0. \tag{12}$$

Proof: This follows directly from Theorem 1 since there exists an universally optimal strategy for this topology. ■

VI. Network 4: Many-to-one interference channel

Finally, we consider the many-to-one interference channel.

A. Direct link knowledge

Theorem 6. *The normalized sum-capacity of a many-to-one interference channel when each of the transmitter knows only its direct link to the receiver is,*

$$\alpha^* = \begin{cases} 1 & \text{if } K = 1, \\ 1/2 & \text{if } K > 1. \end{cases} \tag{13}$$

Proof: When $K = 1$, the interference channel is a point-to-point channel with global information and the same sum-capacity as with the global information can be achieved.

The converse for $K > 1$ follows on the same lines as the proof of Theorem 2 since for $K = 2$, we get a Z-channel in which case the Theorem 2 holds and the same arguments as in Theorem 2 for $K > 2$ apply.

For the achievability, consider the data transfer over two time slots. In the first time-slot, only the K^{th} user transmits while in the second time-slot, all but the K^{th} user transmits data. This way the sum rate of $\sum_{i=1}^{K} n_{ii}/2$ is achieved which is at-least half the sum-capacity with global information and thus normalized sum-capacity of $1/2$ can be achieved. ∎

B. Two hop link knowledge

Theorem 7. *The normalized sum-capacity of a many-to-one interference channel when each of the transmitter knows all the links that are within 2-hop distant from it is,*

$$\alpha^* = \begin{cases} 1 & \text{if } K \leq 2, \\ \frac{K-1}{2K-3} & \text{if } K > 2. \end{cases} \tag{14}$$

Proof: For $K \leq 2$, the normalized sum-capacity is 1 follows from Theorem 1. So, we will only consider $K \geq 3$ in the proof.

We will first prove the converse by contradiction. Suppose that normalized sum rate of $\alpha > \frac{K-1}{2K-3}$ can be achieved. Then, $R_K > \frac{K-1}{2K-3} n_{KK}$ since it has to send at this rate when all other direct channel gains are 0 and are not known to user K. Now, suppose all the channel gains be n. In this case, $R_i < n - \frac{K-1}{2K-3} n = \frac{K-2}{2K-3} n$ for $1 \leq i \leq K-1$. Thus, the sum rate achieved is less than $\left((K-2)\frac{K-2}{2K-3} + 1\right) n = (K-1) n \frac{K-1}{2K-3}$ because $R_{K-1} + R_K \leq n$. Since the optimal sum rate is $(K-1)n$, the sum rate within a factor of $\frac{K-1}{2K-3}$ cannot be achieved thus showing the contradiction. Hence, the normalized sum-capacity cannot be $> \frac{K-1}{2K-3}$.

For the achievability, consider the data transfer over $2K-3$ time slots. In the timeslot i satisfying $1 \leq i \leq K-1$ users i and K transmit. They form an Z-channel and use the optimal strategy for this channel with partial information. In the remaining $K-2$ timeslots, users $1, \cdots, K-1$ transmit at full rate. Let $(R_1, R_2, \cdots, R_K)$ be any point in the global information capacity region. In the i^{th} time-slot where $1 \leq i \leq K-1$, sum rate of atleast $R_i + R_K$ can be achieved while in the remaining $K-2$ timeslots, sum rate of atleast $\sum_{1 \leq i \leq K-1} R_i$ can be achieved. Thus, the sum-capacity with a factor of $\frac{K-1}{2K-3}$ can be achieved. ∎

Note that, since the sum-capacities of many-to-one and one-to-many deterministic interference networks with full-information are known [7], Theorems 4–7 also yield the partial information sum-capacity in each case.

VII. Discussions and Conclusions

In this paper, we considered two forms of partial information at the nodes. The first is when each of the transmitter knows the link gain to only its receiver. This case results in the optimality of scheduling non-interfering users. In a Multiple Access Channel or the up-link problem, this is equivalent to scheduling one user at a time and achieves normalized sum-capacity of $1/K$. For the S-chain, by turning on the even or odd users in the timeslots, we are assigning non-interfering users to transmit in each timeslot thus achieving a normalized sum-capacity of 1/2. In the one-to-many and many-to-one interference channel also, scheduling non-interfering users is optimal. Thus, we provide the first information-theoretic formalism that proves the optimality of MWIS scheduling in these networks.

The second form of partial information considered in the paper is when each transmitter knows link gains of all the links that are at-most two hop distant from it. In this case, we had found in [2] that the connectivities for which there exists a universally optimal strategy are the ones which have all their connected components in one-to-many configuration or in the fully connected configuration. In this paper, we saw that scheduling over such sub-networks is optimal in all four networks, and hence characterized the normalized sum-capacity in each case.

This paper suggests a new scheduling strategy (*sub-network scheduling*) with two hops information at each transmitter. Suppose d time-slots are used and in each time-slot, the network is effectively reduced to a network in which all the connected components are in one-to-many configuration or in fully-connected configuration. If each user is turned on in at-least t time-slots, then normalized sum rate of t/d can be achieved. Note that interference avoidance scheduling uses in each time slot network of single user connected components and is hence a special case. Thus, this better scheduling strategy can be used when two hops of information are available at each transmitter and has been proved optimal in this paper for many channel configurations.

The results in this paper can also be extended to Gaussian interference channels by defining the concept of approximate normalized sum-capacity which is on the lines of approximate universally optimal strategies as defined in [1]. We are currently working on this extension [8].

The problem of optimal strategy with d-hop information at each transmitter in general interference channels is a problem of great importance, and is still open.

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