Optimal Oblivious Reconfigurable Networks

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How do we connect servers so they can communicate?
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How do we route messages along those connections?
Oblivious Reconfigurable Networks (ORNs)

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• Edges reconfigure between each timestep according to a predefined schedule
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• 1-regular directed networks for this talk
  • Results extend to $d$-regular for any constant $d$
Oblivious Reconfigurable Networks
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\begin{array}{c}
\text{a} \\
\downarrow \\
\text{d} \\
\downarrow \\
\text{c} \\
\uparrow \\
\text{b} \\
\uparrow \\
\end{array}
\]
Oblivious Reconfigurable Networks
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Virtual Topology
Oblivious Reconfigurable Networks

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Virtual Topology

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Oblivious Reconfigurable Networks

Virtual Topology

Route $a \rightarrow c$ starting at $t = 1$
Oblivious Reconfigurable Networks

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Oblivious Reconfigurable Networks

Virtual Topology

Route $a \rightarrow c$ starting at $t = 1$
Path has latency $L = 2$
Throughput
Throughput

Demand from $i \to j$ at timestep $t$
Throughput

• A matrix requests throughput $r$ if…
Throughput

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  • Row/column sums $\leq r$

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Throughput

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• An ORN design guarantees throughput $r$ if it can route all matrices requesting throughput $r$ without overloading edges
Main Result

• Given a throughput value $r$, define an ORN design that:
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  • Guarantees throughput $r$
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  • Guarantees throughput \( r \)
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• These objectives are in conflict with each other!
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• These objectives are in conflict with each other!

We fully resolve up to a constant factor!
Throughput v. Latency, $N = 10^{12}$
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Upper Bound
Upper Bound

![Graph showing the upper bound for Design 1](image)

- **Latency** vs **1/Throughput**
  - Logarithmic scale for both axes
  - Data points indicating a decreasing trend as 1/Throughput increases
  - Staircase-like pattern for Latency values

**Legend:**
- **Design 1**
Upper Bound

![Graph showing latency vs. 1/throughput for Design 1 and Design 2]
Lower Bound

![Graph showing latency vs. 1/throughput with a logarithmic scale on the y-axis ranging from $10^0$ to $10^{10}$ and the x-axis from 2 to 18. The graph depicts a decreasing trend as 1/throughput increases.](image)
Lower Bound

![Graph showing latency vs. 1/throughput for uniform demands]
Lower Bound
Ongoing/Future Work

• ORN designs for all $N$ — not just infinitely many
• Semi-oblivious designs and analysis
  • Network still oblivious, but routing may be optimized for traffic
• Practical implementations – Daniel Amir
Thank You! Questions?

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